XXVII. *On the Magnetic Induction of Crystals.*

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INTRODUCTION.

IN repeating Professor FARADAY's experiments, by which he proved that magnetism is a universal agency of nature, to whose influence all bodies are subject, I observed in the year 1847 some strange anomalies, which I attributed to the peculiar structure of the bodies examined. Thus I was led to examine crystalline substances. Between the two poles of a strong electro-magnet I first suspended a plate of tourmaline, then a plate of calcareous spar; and I remarked that these plates, both taken from a polarizing apparatus, were acted upon in an extraordinary way, not dependent on their exterior shape, but solely on their crystalline structure. Since these, my first observations, I have diligently investigated the magnetism of crystals,—this difficult subject of experimental and mathematical inquiry,—guided, as I was, by the conviction that the study of crystals would advance the theory of magnetism as it did previously the theory of light.

The experimental results relating to crystals of the different systems, which, in common with Professor BEER, I have hitherto obtained, are partly published in POGGENDORFF's 'Annalen.' My intention here is not to complete the series of these results. In order to discover the true law of nature, I thought it more important to select, out of the great number of examined crystals, a few proper to be subjected to a closer examination: I chose red ferridcyanide of potassium, sulphate of zinc, and formiate of copper.

I had proposed in the earliest period of these researches an empirical law, intended to connect all observations concerning extraordinary magnetic action exerted on crystals not belonging to the tesseral system. This law, modified subsequently according to new facts observed by Professor FARADAY and myself, does hold with regard to uniaxal crystals having one principal crystallographic axis. Indeed such a crystal, like tourmaline and calcareous spar, when freely oscillating between the two poles, is directed by them exactly in the same way as if the forces resulting from the magnetic action on each particle of the crystal acted upon a fixed line within the crystal—its magnetic axis,—this axis being always forced either into the axial or into the equatorial plane. These conditions will be satisfied by conceiving the least particles of the crystal to be small magnetic needles, becoming magnetic by induction. Crystals of a more complicated structure, like those above mentioned, could not be brought under the same law; for these therefore I supposed *generally* two such magnetic axes. But here I became convinced that the proposed law does not hold when such crystals are examined in all

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directions, and not solely along peculiar ones. Hence nearly two years ago I finally abandoned an hypothesis against which serious doubts had for a long time arisen. For the hypothesis of one or two axes acted upon by the magnet, I substituted another similar hypothesis. In the case of uniaxal crystals, I now conceived an ellipsoid of revolution, consisting of an amorphous paramagnetic or diamagnetic substance, and having within the crystal its principal axis coincident with the principal crystallographic axis. It is easy to verify, that both crystal and ellipsoid, the poles of the magnet not being too near one another, will be directed between them exactly in the same way. In the generalization, an ellipsoid with three unequal axes, having within the crystal a determined direction, must be substituted for the ellipsoid of revolution. In this hypothesis too, we meet with magnetic axes. In the case of uniaxal crystals, the direction I formerly denoted by "magnetic axis" may also be defined as the direction within the crystal round which there is no extraordinary magnetic action. In the general case we get two such directions, which we shall also call "*magnetic axes*," using this name in a different sense from that in which it was employed before. A crystal suspended along either of the two magnetic axes is acted upon like an amorphous body.

According to observation, a crystal under favourable circumstances is directed in the same way as the smallest of its fragments. Hence, according to our new hypothesis, the direction which each of its particles would take, when freely oscillating under the influence of a magnet, may be regarded as determined by an auxiliary ellipsoid. A quite analogous case is that of an amorphous ellipsoid of iron, for instance, with three unequal axes, acted upon by an infinitely distant pole. Here also, according to POISSON's theory, we meet with an auxiliary ellipsoid upon which the pointing of the given one depends. The mode of verifying the existence of such an auxiliary ellipsoid, as well as the laws immediately resulting from it, is exactly the same in both cases. This double verification had the fullest, I may say, an unexpected success. I first proceed to the investigation of the case of POISSON's ellipsoid. Starting from a beautiful theorem lately published by Professor BEER, I was enabled to deduce immediately the analytical expressions, which subsequently I verified in the experimental way. I think this inquiry, in which too I enjoyed Professor BEER's valuable cooperation, will contribute to familiarize experimentalists more and more with the admirable theory, too long neglected, of the French mathematician.

The curious magnetic phenomena I first observed in crystals ten years ago being thus supported by an analytical theory, and the numerical results derived from this theory confirmed by new series of experiments, I take the liberty to lay before the Royal Society an account of my researches. According to the theory of the magnetism of crystals I now propose, the magnetic induction within a crystal is, like the elasticity of the luminiferous ether, determined by means of an auxiliary ellipsoid, which in both cases is similarly placed within the crystal. In both cases there are two fixed directions within it, the two *optic axes* along which there is no double refraction, and the two *magnetic axes*, round which there is no extraordinary magnetic induction. By means of the two optic axes you may obtain the direction of any wave of light entering the crystal, its plane of polarization, and its velocity of propagation. By means of the two magnetic axes, you may obtain, using analytical expressions of nearly the same form, the couple of magnetic forces acting upon a crystal, when suspended between the two poles along any direction whatever, the position of the crystal, and the law of its oscillations *.

I. *On the direction which biaxial crystals assume when suspended between the two poles along different lines, having a determined position with regard to the primitive form of such crystals.*

1. In all the observations I shall describe in this section I made use of a large electro-magnet, excited by six of GROVE'S elements, whose pointed poles were at a distance from each other of 1·6 inch. The crystals, oscillating in the horizontal plane which passes through both poles, and equally distant from these poles, were attached to the double cocoon thread of the torsion balance by means of a hoop, without any other support.

2. *Red ferridcyanide of potassium* ($3K\text{Cy}+\text{FeCy}^3$) is paramagnetic. I observed, in the year 1847, that any fragment of this salt, freely suspended between the two poles of an

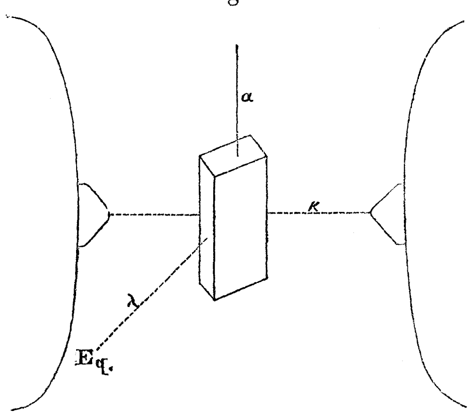
* I think it scarcely necessary here to prove that the theoretical views imputed to me by Professor TYNDALL (Philosophical Transactions, vol. cxlv. p. 2) are not mine, and never have been mine. I never ascribed the phenomena, first observed by myself, to a new force acting upon an ideal line, like the optic axis, quite independent of the paramagnetic condition of the mass of the crystal. Convinced, as I was from the beginning, that there ought to be analogies between the optic and magnetic properties of crystals, I never sought the reason for it anywhere else but in the influence of the crystalline structure on both the luminiferous ether and the magnetic induction. I no more intended to imply a real repulsion or attraction of the optic axes, than the celebrated French philosopher, when he said a beam of light in positive crystals was attracted, in negative ones was repelled, intended to announce a mysterious action emanating in fact from these axes. Such expressions are intended to describe a newly observed fact, but not theoretical views. So also the true meaning of the German words, translated thus, "independent of the paramagnetic or diamagnetic condition of the mass of the crystal," is only this, "whether the mass of the crystal (tourmaline and calcareous spar) be paramagnetic or diamagnetic, the direction of the axis is the same." If, notwithstanding these remarks, there should remain any doubt whatever, I can refer to a paper sent to the Haarlem Society, December 1849, before other philosophers, except Professor FARADAY, had published anything about the magnetic induction of crystals. Starting from mechanical principles, I communicated in this paper a mathematical explanation of what I had observed, for instance, in the case of tourmaline, conceiving this crystal to consist of an infinite number of infinitely small needles, becoming paramagnetic by induction, and being all perpendicular to its axis (see POGGENDORFF'S Annalen, lxxxvi. p. 1). The physical conditions of the question, as there stated, seem to be the same as those which Professor TYNDALL has also adopted in his memoir (p. 45). And though I have recently found reason to modify them, yet there is no trace to be found of the supposition imputed to myself, nor even of "the supposition that the assuming of the axial position proved a body to be magnetic, while the assuming of the equatorial position proved a body to be diamagnetic" (p. 13). When there is an analytical expression, representing the resulting action exerted on a body, now attracted, now repelled, according to distance, it is in most cases mathematically legitimate to speak of two "conflicting forces," by dividing the whole expression into two members, one of which represents an attractive, the other a repulsive force.

electro-magnet, does not point like an amorphous paramagnetic body. The poles being not too near one another, the direction the fragment takes does not depend on its exterior form, but solely on its interior crystalline structure. On account of this strong extraordinary magnetic action, the salt above mentioned appears to be peculiarly fit for exhibiting the phenomena and the laws of this action; the larger the crystals you obtain of this salt, the more easily they may be cut and worked.

We may regard a right prism with a rhombic base as the primitive form of our salt*. There are three crystallographic axes perpendicular to each other, the axis of the prism (α), the shorter diagonal of its base ($\varkappa$), and the longer one (λ).

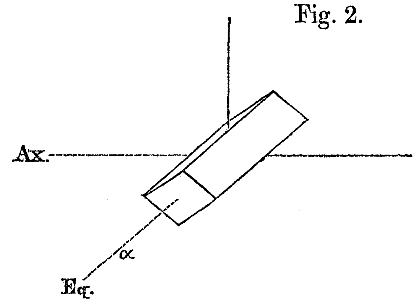
3. A crystal of ferridcyanide of potassium, suspended between the two poles in such a manner that its axis α becomes vertical and therefore its base λ horizontal, sets with energy $\varkappa$ axially (fig. 1). This axis $\varkappa$ remains similarly directed even when we reduce the original prism to a plate, by taking away its obtuse edges. Such a plate appears to be repelled by the two poles like a diamagnetic body, while a plate cut out of the same prism by grinding down its acute edges will be directed, the mode of suspension remaining the same, like a paramagnetic body, as in fact it is.

Fig. 1.



The same prism, suspended in such a manner that its axis α may oscillate between the two poles in the horizontal plane, points equatorially, and seems to be repelled by the poles like a diamagnetic body. We can use in this experiment any natural prism with its summits, whose longest dimension is five or six times greater than its thickness; we may use also a small needle twenty or thirty times as long as it is thick†. In all these cases the oscillating salt finds its position of stable equilibrium by setting itself equatorially. When we reduce the oscillating prism to a plate, by diminishing the dimension of its axis, such a plate, the axis α remaining horizontal, will point axially like an amorphous paramagnetic body.

Fig. 2.



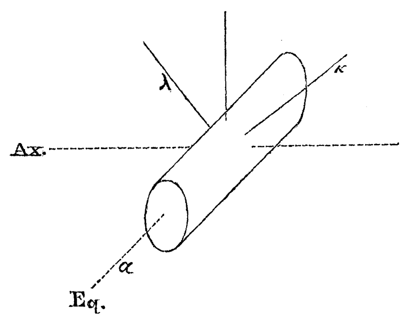
* The primitive form of the salt is disputed. According to the prevalent opinion, we admitted, Professor BEER and myself, in a paper published some years ago, the clinorhombic system, not without some hesitation, as this opinion was neither supported by its magnetic nor its optic properties. I afterwards adopted, in conformity with the new measures recently made by M. SCHABUS of Vienna, the rhombic system. But again, NÖRREMBERG's admirable new arrangement of AMICI's polarizing microscope showing a minute difference between the two systems of rings round the optic axes, not seen in the original apparatus, the question is more doubtful than before.

† Not knowing therefore the extraordinary magnetic action, one would be inclined to range our crystals among diamagnetic bodies, as really has been done.

4. When, always on the supposition of the axis α oscillating horizontally, the shorter diagonal of the rhombic base is vertical, the longer one points axially; when the longer one is vertical, the shorter one points axially. Hence, when any fragment of ferridcyanide of potassium is brought between the two poles, rotating round any one of its three crystallographic axes α, $\varkappa$, λ, this axis being vertical, one of the two remaining axes points axially, and consequently the other equatorially. There are not within the crystal any three other directions enjoying the same property. On this account the three crystallographic axes may also, in the case of our salt, be called *the three axes of paramagnetic induction.* You may distinguish these three axes according to the strength of paramagnetic induction, this induction being in the present case *greatest* along $\varkappa$, *mean* along λ, and *least* along α.

5. When, 1st, we cut out of a crystal a cylinder with a circular base, whose axis is α, we may expect, that such a cylinder horizontally suspended will, when turned round its horizontal axis, retain in all its positions the *equatorial* direction (fig. 3). But the directing power emanating from the poles is not the same in the different positions of the rotating cylinder; it is greatest when $\varkappa$ is horizontal and λ vertical; it is least when λ is horizontal and $\varkappa$ vertical. While the cylinder rotates from the first to the second position, this power gradually diminishes.

Fig. 3.



When, 2nd, we cut out of the same crystal a cylinder whose axis coincides with the shorter diagonal $\varkappa$, such a cylinder, however you may turn it round its axis, supposed horizontal, will always point *axially*. The position agrees with the paramagnetic condition of its substance. But, contrary to this condition, it retains invariably the same position, when, by diminishing its axis, the cylinder is transformed into a circular plate (fig. 4). Such a plate is repelled by the poles with different energy, this energy being greatest when α, and least when λ oscillates horizontally.

Fig. 4.

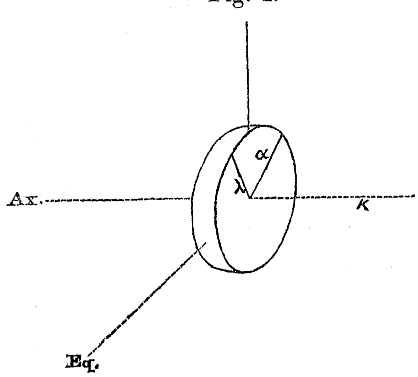


Fig. 5.

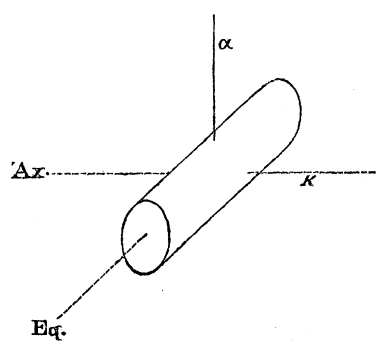
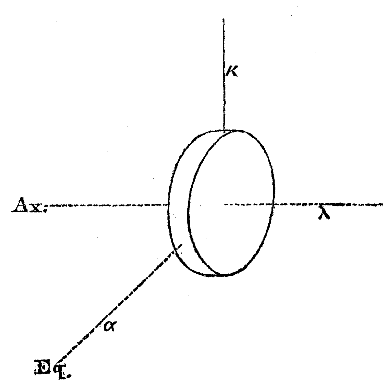


Fig. 6.



When, 3rd, we cut out of the crystal a circular cylinder with its axis parallel to λ, such a cylinder, when horizontally suspended between the two poles, points either equatorially (fig. 5) when α, or axially when $\varkappa$ is vertical. When brought, by turning it round the horizontal axis λ, from the first position to the second, its directive power at first diminishes, till in a certain position it quite *vanishes*; and finally, when the cylinder has passed through this intermediate position, reappears, and increases till the cylinder reaches the second position. The cylinder, when setting its axis equatorially, is directed contrary to the paramagnetic condition of its substance; when axially, in conformity with it. When, as before, we reduce the cylinder to a circular plate, the axis remaining the same, the change of direction, by turning the plate round its horizontal axis, takes place in the same way. But then in the first position the cylinder is directed like a common paramagnetic body, in the second position (fig. 6) like a diamagnetic body.

6. We shall now describe the results obtained by operating with circular cylinders cut out of crystals of ferridcyanide of potassium, in such a way that their axis lies in one of the three principal planes, which, according to the two crystallographic axes they contain, we shall denote by the symbols $\alpha\varkappa$, $\alpha\lambda$, $\varkappa\lambda$. The axis perpendicular to the principal plane in which the axis of the cylinder lies, may be marked, before experimenting, on its bases. In all the following experiments we can replace the circular cylinder by a circular plate having the same axis.

7. I. Let the axis of the cylinder lie in the plane $\varkappa\lambda$, forming with $\varkappa$ any angle ϱ.

a. Whatever may be the angle ϱ, the cylinder, when oscillating round its axis, this axis being vertical, takes always such a position that the plane containing the axes $\varkappa$ and λ becomes axial, the axis α *equatorial* (fig. 7).

Fig. 7.

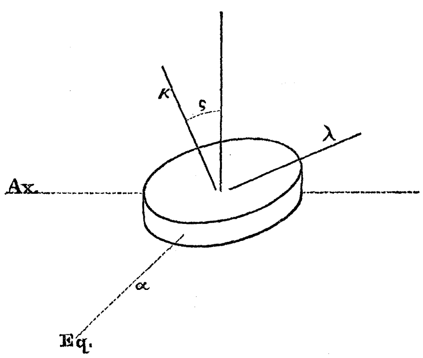


Fig. 8.

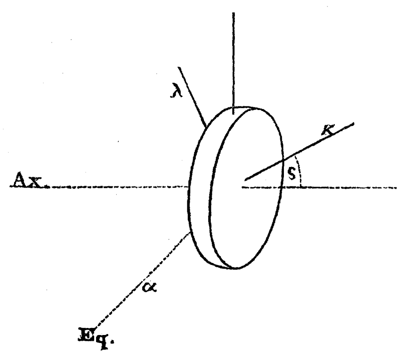
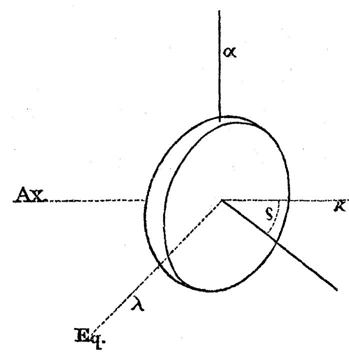


Fig. 9.



b. The cylinder, when suspended in such a way that its axis vibrates horizontally, generally points in an oblique direction, in respect to the axial line. Only in the case where also the axis α is horizontal and therefore the plane $\varkappa\lambda$ vertical, this axis α points equatorially, and therefore the axis of the cylinder *axially* (fig. 8). When, starting from this position, we turn the cylinder round its horizontal axis, the direction of this axis declines from the line joining the two poles; the angle of declination increases till the angle of rotation equals 90°, and therefore the principal plane $\varkappa\lambda$ becomes horizontal (fig. 9); its maximum is ϱ. When the rotation still continues, the axis of the cylinder returns towards the axial line, where it arrives again after a rotation of 180°. From 180° to 360° the same deviation takes place in the same way on the other side of the axial line. Hence, during an entire revolution of the cylinder round its horizontal axis, this axis passes twice through the *axial* line joining the two poles.

8. II. Let the axis of the circular cylinder lie in the principal plane $\alpha\lambda$, making with the crystallographic axis α any angle σ.

a. Whatever may be the angle σ, the cylinder, when oscillating round its axis, this axis being vertical, assumes always such a position that the vertical plane containing the axes α and λ becomes equatorial, the axis $\varkappa$ *axial* (fig. 10).

Fig. 10.

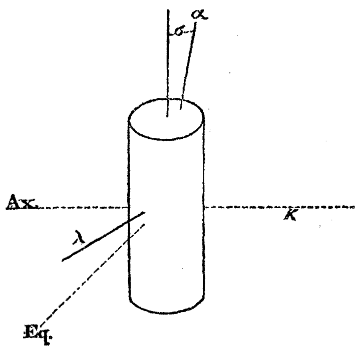


Fig. 11.

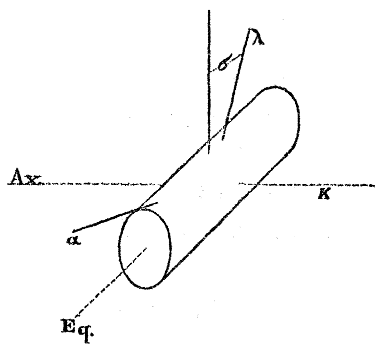
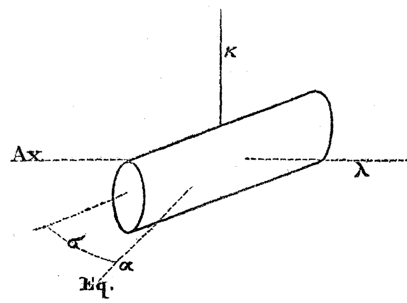


Fig. 12.



b. When suspended in such a way that its axis is horizontal, the cylinder sets it generally neither axially nor equatorially. When it rotates round its horizontal axis, the angle of deviation from the equatorial line varies in a way similar to that in which in the former case the angle of deviation from the axial line varied. There are two positions of the rotating cylinder where its axis is directed *equatorially*. In this case the plane $\alpha\lambda$ becomes vertical (fig. 11). The maximum of the angle of deviation from the equatorial line equals σ, and in this case the plane $\alpha\lambda$ is horizontal (fig. 12). During an entire revolution of the circular cylinder round its horizontal axis, this axis passes twice through the *equatorial* line.

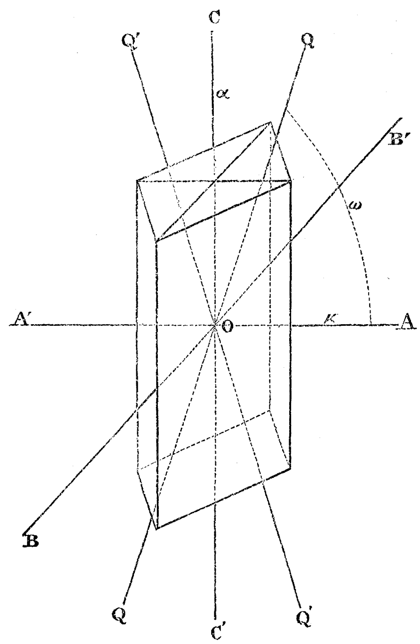
9. III. Let the axis of the circular cylinder lie in the principal plane $\alpha\varkappa$, making any angle τ with the crystallographic axis $\varkappa$.

a. When suspended, its axis being vertical and λ being horizontal, the cylinder will, according to the value of the angle τ, set the axis λ *either axially or equatorially*. By varying the angle τ you will find a certain value of it, which we may denote by ω, for which the circular cylinder will not be directed at all, no more than any amorphous paramagnetic body of the same shape and suspended in the same manner. The angle ω, after some trials, was found to be about 70°. When such a circular cylinder (or plate), unaffected by the magnet, is gently inclined, by turning it round the horizontal axis λ, in one direction and the contrary, the cylinder (or plate) will again take a certain direction; passing through the indifferent state, it will rotate round the vertical line of suspension through an angle of 90°. When $\tau<\omega$, the axis λ points axially; when $\tau>\omega$, equatorially. The directive power of the poles has its two maxima at $\tau=0$ and $\tau=90°$.

From $\tau=0$ till $\tau=\omega$ it gradually decreases, and finally altogether disappears; from $\tau=\omega$ till $\tau=90°$ it increases again.

The angle ω may be taken on either side of the axis $\varkappa$. Hence there are, within crystallized ferridcyanide of potassium, two different directions (QQ and Q′Q′, fig. 13), enjoying each the property that the crystal, when suspended along it between the two poles of a magnet, is acted upon like an amorphous paramagnetic body. It merely points in conformity with its exterior shape; when symmetric round its axis of suspension, it is not directed at all. These two directions, lying in the plane which contains the axis of the primitive prism (α) and the shortest diagonal of its base ($\varkappa$), *i. e.* the axes of greatest and least paramagnetic induction, shall be called *the magnetic axes of the crystal.* The angle between them is about 140°, it is bisected by the crystallographic axis $\varkappa$.

Fig. 13.



b. When the same circular cylinder oscillates between the two poles, its axis being horizontal, this axis points generally in an oblique direction. The cylinder rotating round it, two positions will be found where the axis points either axially or equatorially: it depends upon the value of the angle τ which of these two cases takes place. In this position the crystallographic axis λ lies in the horizontal plane. When the same axis (λ) is vertical, the deviation of the axis of the cylinder from the equatorial or axial line is a maximum, the angle between this axis and the axial line being τ. Between the two mentioned cases, where the axis of the rotating cylinder, according to the value of τ, passes either through the axial or equatorial line, there ought to be an intermediate case, in which the rotating cylinder passes through such a position that the directing power emanating from the poles becomes *uncertain.* It takes place if τ equals about 20° ($=90-\omega$); then the magnetic axis passes through the vertical line. Hence, when the angle τ varies from about $-20°$ to 20°, the axis of the rotating cylinder passes through the axial line; when τ varies from about 20° to 160°, the axis passes through the equatorial line. On this passage the directing power emanating from the poles diminishes when we approach to the intermediate case, where it is zero.

10. In looking over the above-described results obtained by operating on prisms, cylinders, and plates cut out of crystallized ferridcyanide of potassium in various directions with regard to the primitive form of this salt, and suspended in different ways, we may easily remark, that all the observed positions of the crystals between the two poles are exactly the same as those of an ellipsoid made from uncrystallized ferridcyanide or any amorphous paramagnetic substance, and suspended along its different diameters. The three axes of this ellipsoid are to be supposed unequal; they are coincident, according to

their length, with the axis of greatest induction ($\varkappa$), the axis of mean induction (λ), and the axis of least induction (α).

11. *Sulphate of Zinc* ($\dot{\mathrm{Zn}}\,\dddot{\mathrm{S}}$) is a diamagnetic body, showing the extraordinary magnetic action not nearly so strong as the red ferridcyanide of potassium, but strong enough for exhibiting with nicety all the phenomena analogous to those described in the case of the salt before examined. It was especially selected with the intention to examine whether, in regard to the extraordinary magnetic action also, the diamagnetic induction be altogether the contrary of the paramagnetic. This was fully confirmed by experiment.

We may, as in the former case, denote the three crystallographic axes by α, $\varkappa$, and λ, α being the axis of the primitive prism, $\varkappa$ the shorter, λ the longer diagonal of its base.

12. A natural prism oscillating between the two poles, which we always suppose distant enough from each other with regard to the dimensions of the prism, will, when suspended along its axis (α), set equatorially the shorter diagonal of its base ($\varkappa$); when suspended along its shorter diagonal ($\varkappa$), as well as along the longer one (λ), it sets equatorially its axis (α). These directions remain unchanged when the dimensions of the crystal along its crystallographic axes are, in the different modes of suspending, such that the oscillating body, supposed to be an amorphous diamagnetic one, would be directed in the contrary way. Hence, in the case of our salt, the three crystallographic axes may be called *the axes of diamagnetic induction*, α being the axis of *greatest*, $\varkappa$ of *mean*, and λ of *least* induction.

Fig. 14.

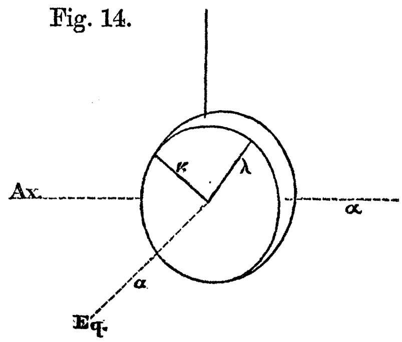


Fig. 15.

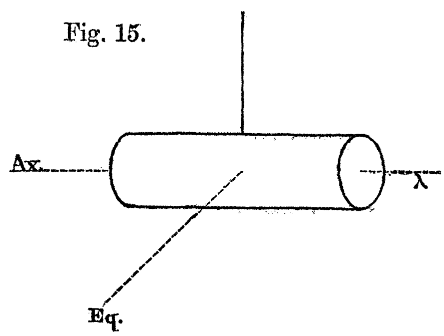


Fig. 16.

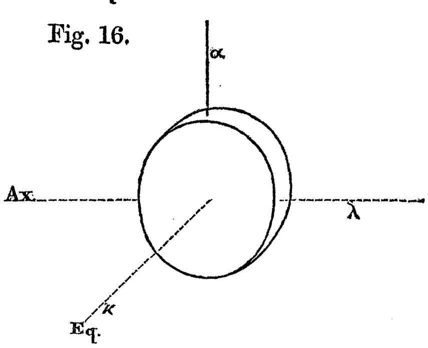
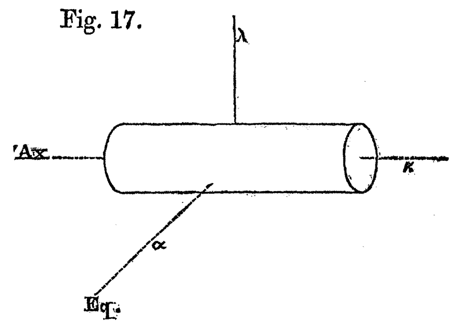


Fig. 17.



13. When we cut out of the crystal a circular plate perpendicular to the axis α, this axis, when oscillating horizontally between the two poles, points equatorially (fig. 14), as in the case of an amorphous paramagnetic body. The directing power emanating from the poles is a maximum if $\varkappa$, a minimum if λ be vertical.

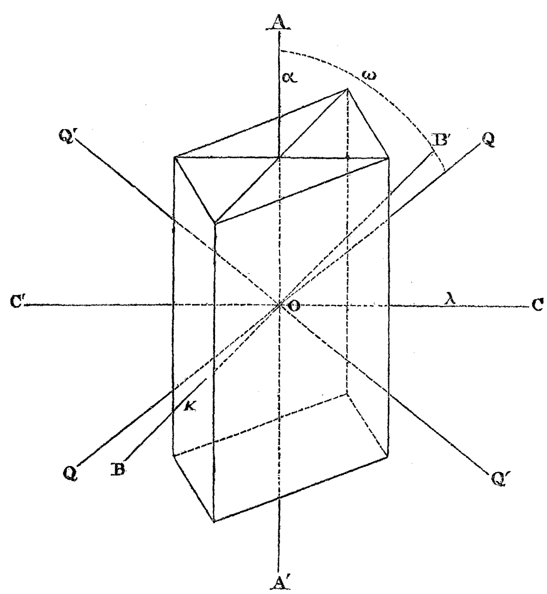
A circular cylinder, whose axis coincides with the longer diagonal (λ), when oscillating horizontally between the poles, sets its axis axially (fig. 15), as an amorphous paramagnetic body of the same shape would do. The directive power is a maximum if $\varkappa$, a minimum if α be vertical.

A circular plate, as well as a circular cylinder, whose axes coincide with the shorter diagonal ($\varkappa$), when suspended with their axes horizontal, set these axes, when rotating round them, either axially or equatorially (figs. 16, 17). There is an intermediate case where no sensible extraordinary magnetic action is observed.

14. A circular cylinder, which also, by diminishing the length of its axis, may be reduced to a circular plate, if its axis do not coincide with one of the three crystallographic axes (α), ($\varkappa$), (λ), points generally obliquely when oscillating with its axis horizontal. It passes, when rotating round its horizontal axis, either through the axial or through the equatorial position. The first case always takes place if its axis lie in the plane $\varkappa\lambda$, the second if it lie in the plane $\alpha\varkappa$. But if the axis lie in the plane $\alpha\lambda$, according to the angles between it and the two axes α, λ, the rotating cylinder passes either through the axial or through the equatorial position.

15. The experiments just described establish the existence, within a diamagnetic crystal also, of two directions, having the property, that the crystal, when suspended along them, is not acted upon by the magnet in an extraordinary way. We may call these directions, as we did in the former case, *the magnetic axes of the crystal.* In order to determine the position of these axes, we proceeded in the following way.

Fig. 18.



First, there was cut out of a large crystal of sulphate of zinc, very easily procurable, a circular plate (nearly 0·5 of an inch diameter and 0·2 of an inch thick) perpendicular to the plane $\alpha\lambda$ and inclined 45° to the base of the prism. On the upper base of the plate was sketched the direction of the shorter diagonal ($\varkappa$). When suspended horizontally (its axis being vertical), the sketched diagonal ($\varkappa$) pointed equatorially, just as a circular plate parallel to the base of the primitive prism would do when horizontally suspended. Secondly, another similar plate was cut out of the crystal, inclined 50° instead of 45° to the base of the primitive form. The new plate horizontally suspended set the shorter diagonal ($\varkappa$) axially. Hence, according to these observations, the angle, with which an indifferent circular plate may be obtained, is between 45° and 50°. It would be difficult to get by this process with certainty closer limits, including the value of this angle. Therefore, the angle between the two magnetic axes is about 95°, and is bisected by the axis of the primitive prism; these axes lie in the plane containing the *acute* edges of this prism (fig. 18).

16. Let us conceive an ellipsoid of amorphous sulphate of zinc or another diamagnetic substance, having within the crystal of this salt its three unequal axes, according to their length, directed along α, $\varkappa$, λ. The positions which the crystal assumes in all the above described modes of suspension, will be imitated by such an ellipsoid when suspended along its corresponding diameters.

17. *Formiate of copper* ($\dot{\mathrm{Cu}}\,\dddot{\mathrm{Fo}}$).—We join to the two examined salts belonging to the same system, one of them being paramagnetic, the other diamagnetic, a third salt, whose primitive form is an oblique prism. There is a plane of symmetry passing through the axis of the prism and the longer diagonal of its rhombic base. The inclination of the axis to the base is 78° 55′; the angles between the lateral faces differ 52′ from a right angle (Heusser): the plane of the base is one of perfect cleavage. This salt, easily crystallizing in large and homogeneous crystals, is paramagnetic, and shows very distinctly the extraordinary magnetic action.

18. At first natural crystals were examined, whose exterior shape had been varied by cleaving them parallel to the base. A horizontal plate bounded by cleavage planes set the symmetrical plane, which, being perpendicular to it, was marked on its upper base, exactly equatorially, even then, when this position did not agree with the position of a similar plate consisting of an amorphous paramagnetic substance. Our plate, when suspended vertically, set the cleavage plane *nearly* equatorially; in this case the plate would rotate through nearly 90°, if it were not crystallized. When turned round the horizontal line perpendicular to its bases, it passed through the equatorial position, its declination from this position remaining always very small.

19. Then, out of a large crystal was cut a circular plate bounded by planes of symmetry, three times as broad as thick. On the surface of the plate, when horizontally suspended and in equilibrium between the two poles, were marked the axial and the equatorial line. Let us denote these two lines, perpendicular to each other, by a and c, the line perpendicular to the plate being denoted by b. The angle within the symmetric plane between the normal to the cleavage plane and a was found to be 3°, taken from the normal towards the obtuse angles of the symmetric plane. The approximate measure of this angle was verified afterwards on different crystals. The same plate oscillating vertically pointed axially when c, equatorially when a was vertical. In the first case only an amorphous paramagnetic body of the same shape would assume the same position. But here also the position of the crystal did not change, after having changed its dimensions in such a way that an amorphous body of the same shape would rotate through an angle of 90° round the vertical axis.

20. The last series of experiments may be described thus: the crystal

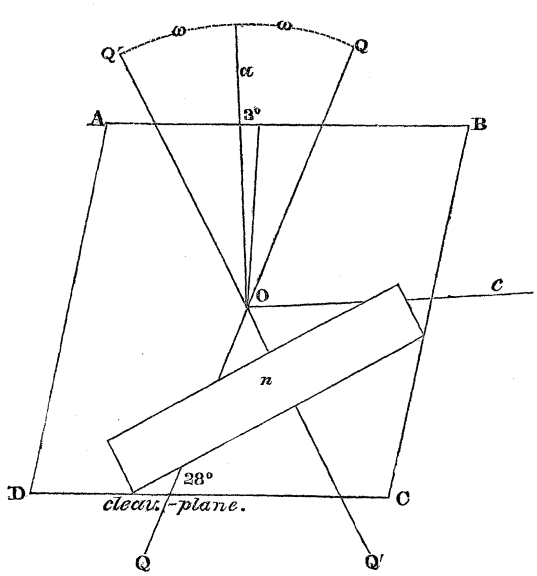
When suspended along a sets axially b;
When suspended along b sets axially a;
When suspended along c sets axially a.

Hence there is proved here also the existence of *three axes of paramagnetic induction*, enjoying the property, that the crystal, when vertically suspended along any one of them, sets one of the remaining two axes axially, the other equatorially. In the case of our salt, the normal to the plane of symmetry is the axis of *mean* induction (*b*). The axes of *greatest* and *least* induction, (*a*) and (*c*), both lie in the plane of symmetry, the first forming an angle of 3° with the normal to the cleavage plane.

We may again conceive an ellipsoid consisting of an amorphous paramagnetic substance, having three unequal axes coincident, according to their length, with *a*, *b*, *c*. Then, in all the different modes of suspension, the crystal will be directed like such an ellipsoid.

21. The *two magnetic axes* are to be sought for within the plane of symmetry containing the greatest and the least axis of induction (*a*), (*c*). Therefore a circular plate was prepared, with a diameter of nearly 0·3 of an inch, bounded by planes perpendicular to the symmetric plane ABCD (fig. 19), and inclined to the cleavage plane (the base DC of the primitive oblique prism) $22\frac{1}{2}°$. Horizontally oscillating, this plate set the plane of symmetry, marked by a line on its upper base, equatorially, as a plate bounded by cleavage planes does. After some trials we prepared a similar plate, II., substituting only an angle of 28° for the above angle of $22\frac{1}{2}°$. Such a plate, horizontally oscillating, was not sensibly directed by the poles. Hence the two magnetic axes, QQ and Q′Q′, of formiate of copper, lie in the symmetric plane, including an angle of about 50°, bisected by the above determined axis of greatest induction (*a*).

Fig. 19.



II. *On finite ellipsoids influenced by an infinitely distant pole.*

22. We have shown that a natural prism of ferridcyanide of potassium, for instance, oscillating between the two poles of a magnet, is kept by them in a fixed position, dependent solely upon the direction of its crystallographic axes with regard to the axis of suspension. It will invariably retain the same position, whatever part we may take away from it, whatever the shape of the remaining fragment may become. From this fact we may conclude that, in the case of this salt, the particles of the influenced crystal do not act sensibly on each other, and hence deduce that the direction of the whole mass of the crystal is plainly defined by the action upon a single one of its molecular particles. According to the above-described experiments, we may infer too, that such a particle is acted upon like a certain amorphous ellipsoid consisting of the substance of the crystal. Again, the action of a magnetic pole upon a single molecule is throughout

analogous to the action of an infinitely distant pole on an ellipsoid of finite dimensions. In both cases the lines of the inductive force are parallel. Thus the solution of the following question coincides with the solution of the question regarding magnecrystallic action:—"To determine the couple of forces acting upon an ellipsoid when influenced by an infinitely distant pole."

23. POISSON gave a complete analytical solution of this question, expressing by means of elliptic functions, which may be calculated in every particular case, the intensity and the direction of the resulting forces, and hence the resulting moment, relating to any given axis of rotation. But the complicated analytical expressions of his formulæ will scarcely allow of deducing from them the law they express. Professor BEER recently succeeded in presenting the results of POISSON'S theory in a most simple and elegant way, making use of an auxiliary ellipsoid, whose three axes, $\left(\frac{1}{a}\right)$, $\left(\frac{1}{b}\right)$, $\left(\frac{1}{c}\right)$, are expressed by elliptic integrals.

24. Let this ellipsoid be intersected in the two points M and M′ by the straight line passing through its centre O and the infinitely distant pole. Construct the two planes touching the ellipsoid in M and M′, and perpendiculars from the centre to these planes, intersecting them in the two points P and P′. Let the distances OM and OM′ be denoted by r, the perpendiculars OP and OP′ by p, the angle between OP and OM by ξ. Finally, determine two points E and E′, lying in OP and OP′, on opposite sides of the centre O, whose distance from the centre equals $\frac{1}{rp}$. Now conceive two ellipsoids, both equal to the given influenced one, and having their axes similarly directed, the first one, with its centre in E, filled with southern, the second, with its centre in E′, filled with northern magnetic fluid. Then the resulting action exerted by the infinitely distant pole, supposed to be a northern one, on the given paramagnetically induced ellipsoid equals a couple of forces, represented by the attraction of the first and the repulsion of the second ellipsoid, both filled with magnetic fluid*.

* The solution given above of POISSON'S problem immediately results from Professor BEER'S communication, which, as it is short, it is but justice to translate here, merely changing, to avoid error, the notation in some cases.

"Let A, B, C be the semi-axes of an ellipsoid, E, electrically influenced by an electric mass, M, infinitely distant along γ, whose action on the unit of volume, filled with the unit of electricity, is Mn. Let $\mu uu'$ be the attraction or repulsion between two infinitely small volumes, u, u', filled with electricity of the density 1, at a distance equal to unity.

"Construct an auxiliary ellipsoid, whose semi-axes $\frac{1}{a^0}, \frac{1}{b^0}, \frac{1}{c^0}$ are directed along the semi-axes A, B, C of the influenced conductor. Take

$$\frac{1}{a^{0\,2}}=\frac{2}{A^2}\int_0^{\pi}d\vartheta\int_0^{\pi}dv\,\frac{\frac{\sin^2\vartheta}{\rho^2}-\frac{\cos^2\vartheta}{A^2}}{\left(\frac{\sin^2\vartheta}{\rho^2}+\frac{\cos^2\vartheta}{A^2}\right)^2}\sin\vartheta,$$

where

$$\frac{1}{\rho^2}=\frac{\cos^2 v}{B^2}+\frac{\sin^2 v}{C^2}.$$

In like manner determine $\frac{1}{b^{0\,2}}$ and $\frac{1}{c^{0\,2}}$ by replacing A by B and A by C. Let r be the radius vector of the

If the induction of the infinitely distant pole, regarded till now to be paramagnetic, become a diamagnetic one, nothing is changed but the sign of the two forces, the first ellipsoid, with its centre in E, being now filled with northern, the second one, with its centre in E′, with southern magnetic fluid.

25. Denoting the value of each force by φ, the resulting moment of rotation is immediately found to be

$$\frac{2\varphi \sin \xi}{pr}, \text{ or } \frac{2\varphi \tan \xi}{r^2} \quad \ldots \ldots \ldots \quad (1.)$$

The axis of this moment, which we shall denote by OR, round which the influenced ellipsoid tends to move, is perpendicular to the plane MOP. The two diameters OM and OR, possessing the property of being axes of the ellipse formed by the intersection of the ellipsoid with the plane passing through them, are *two conjugate axes* of the surface; the relation between the two is a reciprocal one. To any diameter, regarded as one of two such axes, corresponds only one conjugate axis. Therefore the axis round which the body tends to revolve is continually changed, if the given ellipsoid under the influence of the infinitely distant pole freely move round its centre. To any one of the three axes of the auxiliary ellipsoid exceptionally corresponds an infinite number of second conjugate axes, lying all in the principal plane perpendicular to it. Hence the influenced ellipsoid will oscillate continually round such an axis if the infinitely distant pole lie in the conjugate principal plane.

26. When the influenced ellipsoid is only free to rotate round a vertical line passing through its centre—we shall always suppose the infinitely distant pole to lie in the

auxiliary ellipsoid along γ, and construct at its extremity the tangent plane. Let p be the length of the perpendicular from the centre on this plane, and γ' its direction. Let the influenced ellipsoid E move along γ' through the infinitely small distance $\frac{1}{rp}.\sigma$, and denote it in the new position by $E_{\prime}$. By the two ellipsoids E and E′, two infinitely thin sheets are determined, whose acute edges lie in the curve of intersection of E and $E_{\prime}$. One of these two sheets, placed towards M, is exterior to E and interior to $E_{\prime}$; the other, placed oppositely to M, is interior to E and exterior to $E_{\prime}$. Conceive both sheets filled with electricity of the same density, $\frac{\varkappa}{\mu}\frac{M}{\sigma}$, but of a different kind, the electricity of the second sheet being the same as the electricity of M.

"Such is on the surface of the influenced ellipsoid E the distribution of electricity induced by the infinitely distant mass M."—POGGENDORFF's Annalen, xciv. p. 192.

It is well known that the mathematical theory of magnetic induction differs from the theory of electrical induction only by a constant, which POISSON denotes by k; this constant being equal to unity in the last case. In order to apply Professor BEER's construction to magnetic induction, we have only to replace the above defined auxiliary ellipsoid by another, whose semi-axes $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are connected with the former semi-axes $\frac{1}{a^0}, \frac{1}{b^0}, \frac{1}{c^0}$ by the following relations:—

$$\frac{1}{a^2}=2\pi(1-k)+k\frac{1}{a^{0^2}}$$

$$\frac{1}{b^2}=2\pi(1-k)+k\frac{1}{b^{0^2}}$$

$$\frac{1}{c^2}=2\pi(1-k)+k\frac{1}{c^{0^2}}.$$

horizontal plane, which therefore contains OM—we may project OP on the horizontal plane, and denote the angle between this projection OP′ and OM by ξ'. Then the moment of rotation round the vertical axis is found to be

$$\frac{2\varphi \tan \xi'}{r^2}, \quad \ldots \ldots \ldots \ldots \quad (2.)$$

the radius vector r being always directed towards the pole. The horizontal plane is intersected by the auxiliary ellipsoid in an ellipse passing through M. According to simple geometrical relations, the angle ξ' may be defined also to be, in the horizontal plane, the angle between OM and the perpendicular from the centre O on the straight line touching the ellipse in M.

27. In the case of equilibrium, where the moment of rotation disappears,

$$\tan \xi' = 0.$$

This condition is satisfied if one of the two axes of the ellipse lying in the horizontal diametral plane points towards the infinitely distant pole; if the longer axis does, $\frac{1}{r^2}$ is a maximum, the equilibrium therefore an unstable one; if the shorter axis, $\frac{1}{r^2}$ becomes a minimum, the equilibrium a stable one. Hitherto the induction was supposed to be paramagnetic; if it become a diamagnetic one, the unstable equilibrium becomes stable, and *vice versâ*. If the section in the horizontal plane be a circle, the angle ξ' always equals zero, the ellipsoid, therefore, however turned round its vertical diameter, will not move. Hence, the magnetic pole being always situated in the horizontal plane,

An ellipsoid with three unequal axes, oscillating round any of its diameters, supposed to be vertical, when influenced either paramagnetically or diamagnetically by an infinitely distant pole, will be so directed that the auxiliary ellipsoid sets the shorter axis of its horizontal section either axially or equatorially. The two diameters of the auxiliary ellipsoid, perpendicular to its circular sections, are the two magnetic axes of the influenced ellipsoid.

28. In order to verify these results emanating from POISSON'S theory in the case of an ellipsoid with three unequal axes, influenced by an infinitely distant pole, it will be necessary to develop them in the analytical way. The formulæ we shall deduce will find also their immediate application in the case of magnetically induced crystals.

Let us suppose the influenced ellipsoid to rotate round any of its diameters, this diameter being vertical, and the infinitely distant pole lying in the horizontal plane. Its position of equilibrium and the law of its oscillations round the vertical diameter will be determined by the ellipse in which the horizontal plane intersects the auxiliary ellipsoid. This ellipsoid is represented in the ordinary way by the equation

$$a^2x^2 + b^2y^2 + c^2z^2 = 1, \quad \ldots \ldots \ldots \ldots \quad (3.)$$

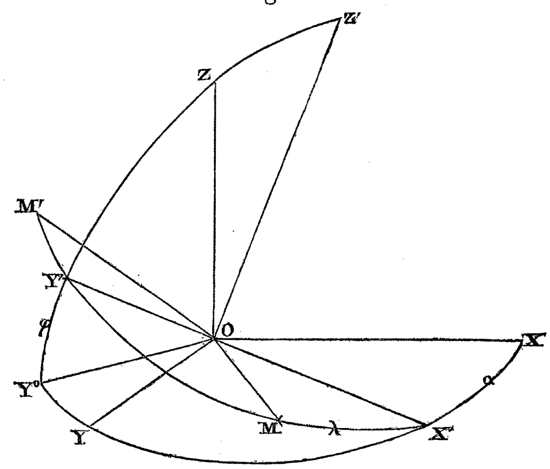
where

$$a^2 > b^2 > c^2.$$

The greatest axis of the influenced and the least of the auxiliary ellipsoid lie along

OX (fig. 20), the mean axis of both ellipsoids along OY, the least axis of the first and the greatest of the second along OZ. We may determine the horizontal plane X′OY′, passing through the common centre of both ellipsoids, by two angles, α and φ; α being the angle between the axis OX and the line OX′ in which the plane XOY is intersected by the plane X′OY′, and φ the angle between the two planes. This determination admits no ambiguity when we conceive the plane XOY to pass from its original position by a double rotation into the horizontal plane X′OY′; rotating first round OZ through the angle α, taken from OX towards OY, till OX coincides with OX′ and OY with a line we shall denote by OY⁰; rotating secondly round OX′ through the angle φ, taken in the plane ZOY⁰, from OY⁰ towards OZ, till OY⁰ coincides with OY′ and OZ with OZ′. The three new axes, OX′, OY′, OZ′, are perpendicular to each other, as the primitive ones are.

Fig. 20.



The equation of the ellipse in the horizontal plane X′OY′, referred to the axes of coordinates OX′ and OY′, is immediately obtained, when in the equation (3.), by means of the following relations,

$$x=x' \cos\alpha - y' \sin\alpha \cos\varphi,$$
$$y=x' \sin\alpha + y' \cos\alpha \cos\varphi,$$
$$z=y' \sin\varphi,$$

x, y and z are replaced by x' and y'. It becomes

$$\varrho x'^2 + 2\sigma x'y' + \tau y'^2 = 1, \quad \ldots \ldots \quad (4.)$$

by putting, for brevity,

$$\left.\begin{aligned} a^2\cos^2\alpha + b^2\sin^2\alpha &= \varrho \\ -(a^2-b^2)\sin\alpha\cos\alpha\cos\varphi \quad . &= \sigma \\ (a^2\sin^2\alpha + b^2\cos^2\alpha)\cos^2\varphi + c^2\sin^2\varphi &= \tau. \end{aligned}\right\} \quad \ldots \ldots \quad (5.)$$

Denoting the two semi-axes OM and OM′ of this ellipse by $\frac{1}{a'}$ and $\frac{1}{b'}$, and the two angles, MOX′ and M′OX′, between them and the axis OX′ by λ and $(\lambda + \frac{1}{2}\pi)$, we get the well-known equations—

$$(a'^2 - b'^2)^2 = (\varrho - \tau)^2 + 4\sigma^2, \quad \ldots \ldots \quad (6.)$$

$$\tan 2\lambda = -\frac{2\sigma}{\varrho - \tau}. \quad \ldots \ldots \quad (7.)$$

29. From these two equations we may first deduce the following one,

$$(a'^2 - b'^2) = \pm \frac{2\sigma}{-\sin 2\lambda},$$

whence, by substitution,

$$(a'^2-b'^2)=\pm(a^2-b^2)\frac{\sin 2\alpha}{\sin 2\lambda}\cos\varphi. \qquad (8.)$$

30. Again, the formula (7.) may be expanded thus:—

$$\tan 2\lambda=-\frac{\sin 2\alpha\cos\varphi}{\cos 2\alpha+k\sin^2\varphi}, \qquad (9.)$$

where

$$k=\frac{a^2\sin^2\alpha+b^2\cos^2\alpha-c^2}{a^2-b^2}=\sin^2\alpha+\frac{b^2-c^2}{a^2-b^2}.$$

Denoting the angle between the two magnetic axes, perpendicular to the circular sections of the auxiliary ellipsoid (3.), by 2ω (fig. 21), we obtain

$$\frac{b^2-c^2}{a^2-b^2}=\tan^2\omega, \qquad \frac{b^2-c^2}{a^2-c^2}=\sin^2\omega, \qquad \frac{a^2-b^2}{a^2-c^2}=\cos^2\omega, \qquad (10.)$$

whence

$$k=\sin^2\alpha+\tan^2\omega. \qquad (11.)$$

Fig. 21.

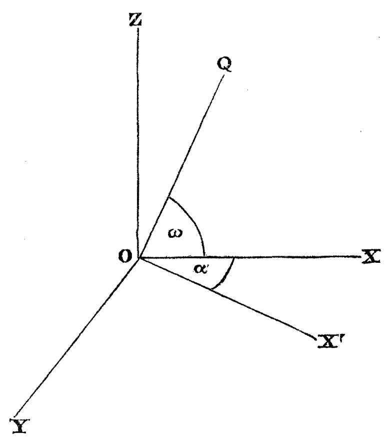


Fig. 22.

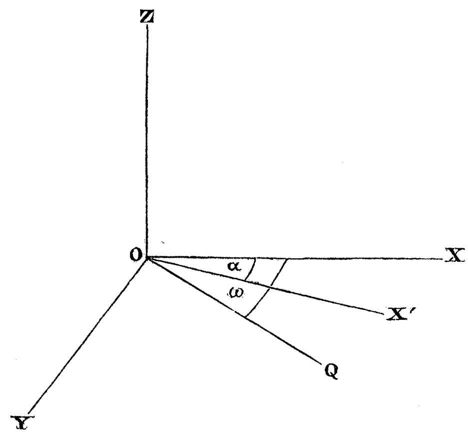
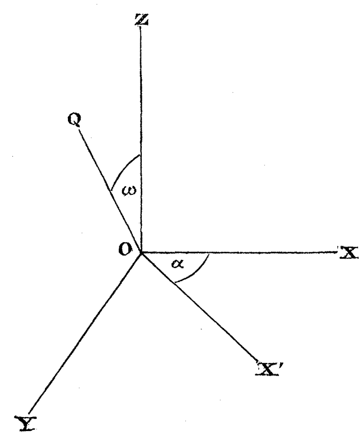


Fig. 23.



31. It will be in some cases more convenient to refer the horizontal intersecting plane to another principal section of the auxiliary ellipsoid. Hitherto we have supposed—and so we shall do again in the following articles—the shortest axis $\frac{1}{a}$, the mean $\frac{1}{b}$, and the longest $\frac{1}{c}$ to coincide with OX, OY, and OZ. Now let $\frac{1}{a}$ fall, as before, on OX, but $\frac{1}{c}$ on OY, and $\frac{1}{b}$ on OZ, and accordingly let α be taken in the plane containing the shortest and the longest axis (fig. 22), from the former towards the latter. In this case b is to be replaced by c, and *vice versâ*; therefore

$$k=\sin^2\alpha-\sin^2\omega. \qquad (12.)$$

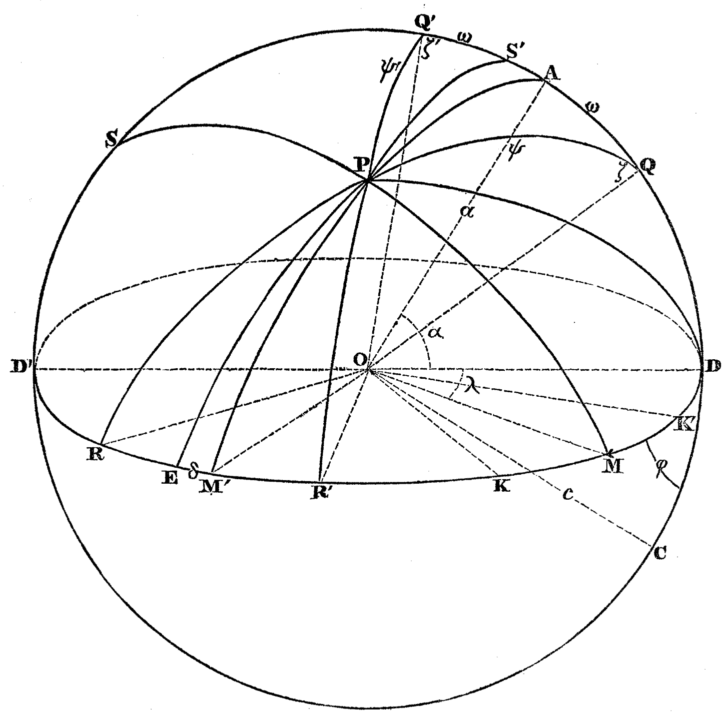
32. When, thirdly, $\frac{1}{a}$ falls on OZ, $\frac{1}{b}$ on OX, and $\frac{1}{c}$ on OY, and accordingly α is taken in the plane YOX (fig. 23), from the mean axis $\frac{1}{b}$ towards the longest $\frac{1}{c}$, we get

$$k=\sin^2\alpha-\cos^2\omega. \qquad (13.)$$

33. By means of the formulæ (9.) and (11.)—(13.), we can determine the position of the influenced ellipsoid, if by the angles α and φ its horizontal section be fixed, with regard to any of its three principal planes. The two axes of the elliptic section of the auxiliary ellipsoid in the same horizontal plane, determined both by λ, will point axially and equatorially. The same two axes may be found also by the most simple geometrical construction.

34. Let the axis $\frac{1}{a}$ fall on OA, the axis $\frac{1}{c}$ on OC; let OQ and OQ′ (fig. 24) be the two magnetic axes lying in the plane containing OA and OC, therefore AQ=AQ′=ω.

Fig. 24.



Let DMD′ be the horizontal plane, AD being α and MDC=φ; let P be the pole of this plane, whence OP the vertical axis of suspension. Let OE be the projection of OA on the horizontal plane, OR and OR′ the projections of the two magnetic axes OQ and OQ′. Let OK and OK′ in the same plane be the traces of the two circular sections of the auxiliary ellipsoid, and OM and OM′ the two semi-axes of the horizontal elliptic section, pointing axially and equatorially.

The two semidiameters (OK and OK′) in which any elliptical section of the ellipsoid is intersected by its two circular sections, are both equal to the mean semi-axis $\frac{1}{b}$. The two semidiameters (OR and OR′) of the same elliptical section, perpendicular to OK and OK′, are likewise equal to one another. Hence we shall find the two axes, OM and OM′, of this section, here supposed horizontal, by bisecting the angle RR′ and its supplementary angle.

35. This construction of the two axes OM and OM′ is easy to execute on a given sphere; we may also without difficulty transform it into analytical expressions.

Considering the three rectangular spherical triangles AED, QRD, and Q′R′D, the angle ADE being $\pi-\phi$, we get

$$\left.\begin{aligned}\tan \mathrm{DE} &= -\tan \alpha \cos \phi, \\ \tan \mathrm{DR} &= -\tan (\alpha-\omega) \cos \phi, \\ \tan \mathrm{DR}' &= -\tan (\alpha+\omega) \cos \phi,\end{aligned}\right\} \quad \ldots \ldots \quad (14.)$$

whence

$$\left.\begin{aligned}\lambda+\pi=\tfrac{1}{2}(\mathrm{DR}+\mathrm{DR}')=\mathrm{DM}', \\ \delta=\mathrm{DE}-\mathrm{DM}',\end{aligned}\right\} \quad \ldots \ldots \quad (15.)$$

denoting by δ the angle between OE, the projection of OA on the horizontal plane, and OM′ pointing within this plane axially or equatorially.

36. The following relations too will afterwards be employed. From the two triangles QRD and Q′R′D, just considered, we deduce also,

$$\cos \mathrm{DR} = -\frac{\cos(\alpha-\omega)}{\sin \psi}, \qquad \sin \mathrm{DR} = \sin \zeta \sin (\alpha-\omega),$$

$$\cos \mathrm{DR}' = -\frac{\cos(\alpha+\omega)}{\sin \psi'}, \qquad \sin \mathrm{DR}' = \sin \zeta' \sin (\alpha+\omega),$$

denoting by ψ and ψ' the angles PQ and PQ′, between the two magnetic axes (OQ and OQ′) and the vertical line OP, and by ζ and ζ' the angles DQP and DQ′P. Hence

$$\sin 2\lambda = -\sin (\mathrm{DR}+\mathrm{DR}') = \frac{\sin \zeta}{\sin \psi'} \sin (\alpha-\omega) \cos (\alpha+\omega) + \frac{\sin \zeta'}{\sin \psi} \cos (\alpha-\omega) \sin (\alpha+\omega).$$

But from the triangles DQP and DQ′P we get, remembering that $\mathrm{PDQ}=\frac{1}{2}\pi-\phi$ and $\mathrm{PD}=\frac{1}{2}\pi$,

$$\sin \zeta \sin \psi = \sin \zeta' \sin \psi' = \cos \phi,$$

whence

$$\frac{\sin \zeta}{\sin \psi'} = \frac{\sin \zeta'}{\sin \psi} = \sqrt{\frac{\sin \zeta \sin \zeta'}{\sin \psi \sin \psi'}} = \frac{\cos \phi}{\sin \psi \sin \psi'};$$

therefore, expanding the last equation,

$$\sin \psi \sin \psi' = \frac{\sin 2\alpha \cos \phi}{\sin 2\lambda}. \quad \ldots \ldots \quad (16.)$$

Reverting to the equation (8.), in which here, according to art. (31), b^2 is to be replaced by c^2, we may write it now thus:—

$$(a'^2-b'^2)=(a^2-c^2) \sin \psi \sin \psi'. \quad \ldots \ldots \quad (17.)$$

37. By means of the angle ω we have determined the position of the influenced ellipsoid: this position, reciprocally, being determined by observation in any particular case, we can find the angle ω. We may use for this purpose the formulæ (9.) and (11.)—(13.). But a simple geometrical consideration will equally lead us to the determination of the value of ω.

The two vertical planes SPM and S′PM′, containing the two axes of the horizontal section, and the two planes QPR and Q′PR′ bisecting them, constitute what is called a system of four harmonic planes. Such a system is intersected by any plane in four harmonic lines. OS, OS′ and OQ, OQ′ are therefore four harmonic lines, whence the angle 2ω between OQ and OQ′ being bisected by OA,

$$\tan\eta\tan\eta'=\tan^2\omega, \quad \ldots\ldots\ldots\ldots \quad (18.)$$

denoting the angles OS and OS′ by η and η'. Again, in the triangle PDS, observing that $\text{PDS}=\frac{1}{2}\pi-\phi$, $\text{DPS}=\pi-\lambda$, $\text{DS}=\eta+\alpha$, we get

$$\left.\begin{aligned}\tan(\eta+\alpha)&=-\frac{\tan\lambda}{\cos\phi},\\ \text{whence}\qquad \tan(\eta'+\alpha)&=\frac{\cot\lambda}{\cos\phi}.\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (19.)$$

The position of the horizontally oscillating plane being determined, within the influenced ellipsoid, by the two angles α and ϕ, and the two directions within this plane pointing axially and equatorially by λ, the last two equations furnish the values of η and η', whence, by means of (18.), we obtain the value of ω, and consequently the directions of the two magnetic axes.

38. We have hitherto considered as known the direction of the three axes of the auxiliary ellipsoid. Such is the case in the question of a given ellipsoid, influenced by an infinitely distant magnetic pole, where these three axes are coincident with the three axes of the influenced ellipsoid. But, when treating on the magnetic induction of crystals, we shall meet with questions where the direction of the axes of the auxiliary ellipsoid is to be determined by experiment. If a given ellipsoid be suspended along any diameter, we can find the two axes of the horizontal section of the auxiliary ellipsoid, these axes pointing, one axially, the other equatorially. The new question therefore is a geometrical one, "To determine the three axes of an ellipsoid, knowing the two axes of each of its sections," and may be resolved in the following way.

Let the given influenced ellipsoid revolve round any one of its diameters, supposed to lie in the horizontal plane, and mark in each of its positions the axial as well as the equatorial line. These two lines, two conjugate axes of the auxiliary ellipsoid, will describe during one revolution a conic surface of the third order, containing the three axes of the auxiliary ellipsoid; for these axes will successively pass through the horizontal plane, and then point either axially or horizontally. Hence two such conic surfaces will determine the three axes of the auxiliary ellipsoid.

39. Let any two conjugate axes of the auxiliary ellipsoid be represented by

$$\begin{aligned}x&=gz, \qquad & x&=g'z,\\ y&=hz, & y&=h'z,\end{aligned}$$

while this ellipsoid is always represented by

$$a^2x^2+b^2y^2+c^2z^2=1.$$

Then

$$\begin{aligned}gg'+\ hh'+1&=0,\\ a^2gg'+b^2hh'+c^2&=0,\end{aligned}$$

whence

$$\left.\begin{aligned} gg'&=\frac{b^2-c^2}{a^2-b^2}=\tan^2\omega, \\ hh'&=-\frac{a^2-c^2}{a^2-b^2}=-\frac{1}{\cos^2\omega} \end{aligned}\right\} \quad \ldots \ldots \ldots \quad (20.)$$

The two conjugate axes, when lying in the above-mentioned conic surface, are contained in a plane passing also through the horizontal diameter round which the given ellipsoid revolves. This diameter being represented by

$$x=mz,$$
$$y=nz,$$

we get, therefore,

$$(g'-g)n+(h-h')m+(gh'-hg')=0.$$

Eliminating g and h' by means of (20.), and putting $\frac{x}{z}$ and $\frac{y}{z}$ instead of g and h, we obtain the following equation:

$$\frac{(nx-my)}{z}\cos^2\omega+\frac{(y-nz)}{x}\sin^2\omega+\frac{x-mz}{y}=0, \quad \ldots \quad (21.)$$

representing the conic surface of the third order.

40. Reverting to the rotation of the influenced ellipsoid, we immediately obtain the instantaneous axis of rotation. The given diameter OM passing, when prolonged, through the infinitely distant pole, and this axis being two conjugate axes of the auxiliary ellipsoid, each of them is determined by the other by means of (20.). Denoting the angles between OM and the three axes of coordinates OX, OY, OZ by μ, ν, ϱ, and those between the instantaneous axis of rotation and the three same axes of coordinates by μ', ν', ϱ', we have

$$-\frac{\cos\mu\cos\mu'}{\sin^2\omega}=\cos\nu\cos\nu'=-\frac{\cos\varrho\cos\varrho'}{\cos^2\omega}. \quad \ldots \quad (22.)$$

41. The absolute moment of rotation found to be

$$\frac{2\varphi\tan\xi}{r^2}$$

may easily be expanded: ξ being the angle between the diameter OM passing through M, whose coordinates may be denoted by x, y, z, and the perpendicular to the plane touching the auxiliary ellipsoid in this point, we get by well-known formulæ,

$$\tan^2\xi=[(a^2-b^2)xy]^2+[(a^2-c^2)xz]^2+[(b^2-c^2)yz]^2=(a^2-c^2)^2[x^2y^2\cos^2\omega+x^2z^2+y^2z^2\sin^2\omega],$$

whence

$$\left(\frac{\tan\xi}{r^2}\right)^2=(a^2-c^2)[\cos^2\mu\cos^2\nu\cos^2\omega+\cos^2\mu\cos^2\varrho+\cos^2\nu\cos^2\varrho\sin^2\omega];$$

and by eliminating $\cos^2\nu$ by means of

$$\cos^2\mu+\cos^2\nu+\cos^2\varrho=1,$$

and by reducing,

$$\left(\frac{\tan\xi}{r^2}\right)^2=\tfrac{1}{4}(a^2-c^2)^2[\sin^2 2\mu+\sin^2\omega(\sin^2 2\varrho-\sin^2 2\mu)]. \quad \ldots \quad (23.)$$

When the infinitely distant pole falls successively within each of the three principal sections of the auxiliary ellipsoid, the following resulting moments of rotation are obtained:—

$$\varphi(a^2-b^2)\sin 2\mu, \qquad \varphi(a^2-c^2)\sin 2\mu, \qquad \varphi(b^2-c^2)\sin 2\nu.$$

42. Now let us suppose the given ellipsoid to rotate round its vertical diameter, the horizontal plane being determined by any two angles α and φ. Let the elliptical section of the auxiliary ellipsoid within the horizontal plane (4.) be represented by

$$a'^2x^2+b'^2y^2=1,$$

its shorter semi-axis $\frac{1}{a}$ lying in the axis of abscissæ. Let r' be the length of the semi-diameter OM of this elliptical section passing, if prolonged, through the infinitely distant pole, and x' and y' the coordinates of its extremity, M. Then

$$\tan\xi'=(a'^2-b'^2)x'y',$$

whence the moment of rotation round the vertical axis

$$\frac{2\varphi\tan\xi'}{r'^2}=2\varphi(a'^2-b'^2)\frac{x'y'}{r'^2}=2\varphi(a'^2-b'^2)\sin\vartheta\cos\vartheta, \quad \dots \quad (24.)$$

the angle between the radius vector r' and the shorter axis $\frac{1}{a'}$ being ϑ.

43. The oscillations of the influenced ellipsoid, when infinitely small, may easily be analytically determined. The ellipsoid, supposed to be paramagnetically induced, is in stable equilibrium when the shorter semi-axis $\left(\frac{1}{a'}\right)$ of the elliptical section of the auxiliary ellipsoid within the horizontal plane points towards the infinitely distant pole. When rotated through an infinitely small angle, this angle being the angle ϑ between r' and the axis $\left(\frac{1}{a'}\right)$, the corresponding moment of rotation becomes

$$2\varphi(a'^2-b'^2)\vartheta. \quad \dots \quad (25.)$$

This expression will remain unchanged when the paramagnetic induction becomes a diamagnetic one; φ becoming, in this case, negative and the longer axis $\left(\frac{1}{b'}\right)$, instead of the shorter $\left(\frac{1}{a'}\right)$, directed towards the infinitely distant pole.

We obtain therefore, in both cases,

$$\frac{d^2\vartheta}{dt^2}=\frac{2\varphi(a'^2-b'^2)}{\text{MK}^2}\vartheta,$$

denoting the mass of the influenced ellipsoid by M, and its moment of inertia with regard to the vertical axis by MK^2. Consequently the ellipsoid, under the influence of the infinitely distant pole, oscillates like a common pendulum. Denoting the time of one oscillation by Θ, we get, in the ordinary way, by integration,

$$\Theta^2=\frac{\text{MK}^2\pi^2}{2\varphi(a'^2-b'^2)}. \quad \dots \quad (26.)$$

44. When we suppose the three semi-axes A, B, C of the influenced ellipsoid to be successively vertical, the corresponding values of K^2 become

$$\tfrac{1}{5}(\text{B}^2+\text{C}^2)=\text{K}_{,}^2, \quad \tfrac{1}{5}(\text{A}^2+\text{C}^2)=\text{K}_{,,}^2, \quad \tfrac{1}{5}(\text{A}^2+\text{B}^2)=\text{K}_{,,,}^2,$$

whence, in the general case, α^0, β^0, γ^0 being the angles between the vertical axis of rotation and the three semi-axes A, B, C,

$$K^2=K_{,}^2\cos^2\alpha^0+K_{,,}^2\cos^2\beta^0+K_{,,,}^2\cos^2\gamma^0.$$

The time corresponding to one oscillation round A, B, C being denoted by $\Theta_{,}$, $\Theta_{,,}$, $\Theta_{,,,}$, we have

$$\left.\begin{aligned}\Theta_{,}^2&=\tfrac{1}{10}\pi^2\frac{M}{\varphi}\cdot\frac{B^2+C^2}{b^2-c^2},\\ \Theta_{,,}^2&=\tfrac{1}{10}\pi^2\frac{M}{\varphi}\cdot\frac{A^2+C^2}{a^2-c^2},\\ \Theta_{,,,}^2&=\tfrac{1}{10}\pi^2\frac{M}{\varphi}\cdot\frac{A^2+B^2}{a^2-b^2}.\end{aligned}\right\}\quad\quad(27.)$$

Remembering the relations (10.), we find, by division,

$$\left.\begin{aligned}\frac{\Theta_{,,}^2}{\Theta_{,,,}^2}&=\frac{A^2+C^2}{A^2+B^2}\cdot\cos^2\omega,\\ \frac{\Theta_{,,}^2}{\Theta_{,}^2}&=\frac{A^2+C^2}{B^2+C^2}\cdot\sin^2\omega,\\ \frac{\Theta_{,,,}^2}{\Theta_{,}^2}&=\frac{A^2+B^2}{B^2+C^2}\cdot\tan^2\omega;\end{aligned}\right\}\quad\quad(28.)$$

and likewise, according to (17.),

$$\frac{\Theta_{,,}^2}{\Theta^2}=\frac{K_{,,}^2}{K^2}\cdot\frac{a'^2-b'^2}{a^2-c^2}=\frac{K_{,,}^2}{K^2}\sin\psi\sin\psi',\quad\quad(29.)$$

ψ and ψ' being, as before, the angles between the vertical axis of rotation and the two magnetic axes.

45. Eliminating, finally, ω from any two of the three equations (28.), we obtain

$$\frac{A^2+B^2}{\Theta_{,,,}^2}+\frac{B^2+C^2}{\Theta_{,}^2}=\frac{A^2+C^2}{\Theta_{,,}^2}.\quad\quad(30.)$$

46. With regard to the magnetic induction of crystals, the following geometrical considerations will not appear without some interest.

The auxiliary ellipsoid, whose equation in rectangular coordinates is

$$a^2x^2+b^2y^2+c^2z^2=1,\quad\quad(3.)$$

may be replaced by another one represented by

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1.\quad\quad(31.)$$

The new ellipsoid, whose axes are a, b, c, may be called the *first* auxiliary ellipsoid; the ellipsoid hitherto made use of, the *second* auxiliary one. The two ellipsoids are polar surfaces with regard to a concentric sphere whose radius equals unity. The two magnetic axes hitherto defined to be the two perpendiculars to the circular sections of the second auxiliary ellipsoid, may be defined also, with regard to the first auxiliary ellipsoid, to be the axes of the circumscribed circular cylinders. The resulting couple $\frac{2\varphi\sin\xi}{rp}$

may also be written thus,

$$2\varphi p_0 r_0 \sin\xi = \varphi r_0^2 \sin 2\xi,$$

r_0 being the radius vector OM_0 of the first auxiliary ellipsoid lying along $p(=OP)$, and $p_0(=OP_0)$ the perpendicular to the plane touching it in the extremity of this radius vector and coinciding with $r(=OM)$.

47. Let ϱ $(=OE)$ be equal to $r_0 p_0$ and directed along OP and OM_0; let x_0, y_0, z_0 be the coordinates of M_0, and α_0, β_0, γ_0 the angles between $\varrho(=OE)$ and the three axes of coordinates; then

$$\left(\frac{1}{\varrho}\right)^2 = \frac{1}{r_0^2}\left(\frac{x_0^2}{a^4}+\frac{y_0^2}{b^4}+\frac{z_0^2}{c^4}\right) = \frac{\cos^2\alpha_0}{a^4}+\frac{\cos^2\beta_0}{b^4}+\frac{\cos^2\gamma_0}{c^4}.$$

This relation shows that the point E falls on the surface of a new ellipsoid, which may be called the *ellipsoid of induction.* Its three semi-axes are a^2, b^2, c^2. Therefore the ellipsoid of induction and the concentric sphere, whose radius is equal to unity, are two polar surfaces, with regard to the first auxiliary ellipsoid, the polar plane of the point E touching the sphere in a point K, in which the sphere is intersected by OM_0.

By means of the ellipsoid of induction, which may be represented by

$$\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}+\frac{z^2}{\gamma^2}=1, \quad (32.)$$

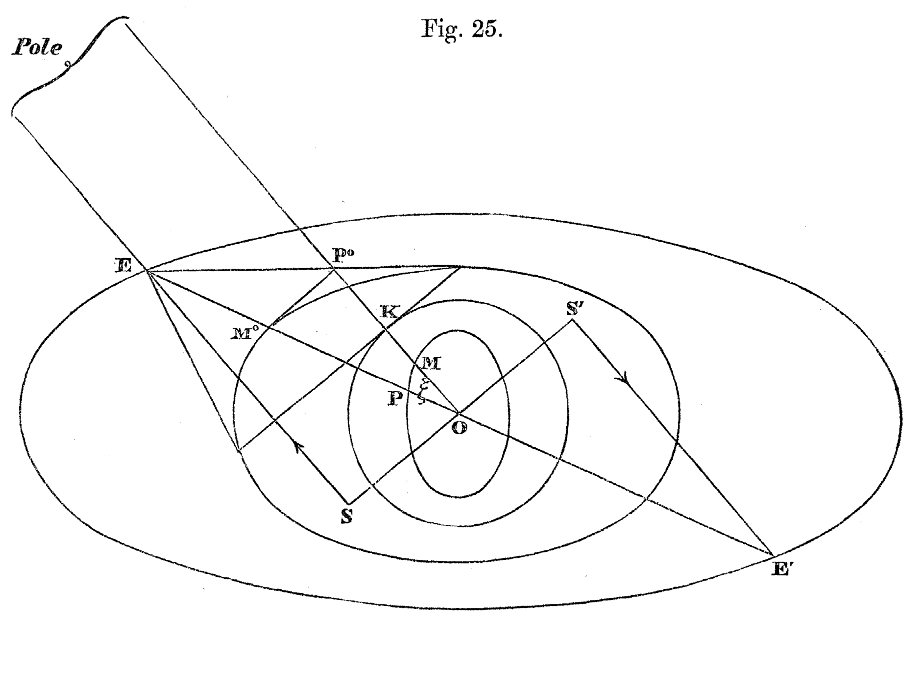
we can completely resolve the proposed question, replacing in all the former formulæ a^2, b^2, c^2 by α, β, γ. Thus, for instance,

$$\cos^2\omega = \frac{\alpha-\beta}{\alpha-\gamma}, \quad \cos 2\omega = \frac{\alpha+\gamma-2\beta}{\alpha-\gamma},$$

whence the two magnetic axes are known.

48. With reference to the relations between the different ellipsoids and the sphere above mentioned, we easily obtain various constructions of POISSON's problem, among which I select the following one (fig. 25).

Fig. 25.



In the general case, where the influenced ellipsoid is free to rotate round its centre, construct the planes touching the sphere in the points K and K′, in which it is intersected by the radius vector OM passing, when prolonged, through the infinitely distant magnetic pole; and let the conjugate poles of these planes, with regard to the first auxiliary ellipsoid, be E and E′. Then EOE′ is a diameter of the ellipsoid of induction; its projection SOS′ on any plane perpendicular to OM (containing, for instance, the centre O) represents the absolute moment of rotation (putting $\varphi=1$), and the line within the same plane, perpendicular to the projection SOS′, is the instantaneous axis of rotation.

49. In order to verify the results emanating from Poisson's theory, I appealed to the known ability of M. Fessel of Cologne, to turn, out of a homogeneous piece of soft iron, two ellipsoids with unequal axes, the position of the centres and the length of the diameters of one set of their circular sections having been previously calculated. The ratio of the squares of the three axes in both ellipsoids was fixed as follows:—

$$A^2 : B^2 : C^2 = 400 : 160 : 100.$$

According to this ratio, both sets of circular sections were perpendicular to each other, as were also the diameters perpendicular to both sets of circular sections. The longest axis, 2A, of the first ellipsoid was 3·16 inches; the second ellipsoid had only half the dimensions of the first.

50. 1st. The first ellipsoid was attached at the extremities of its longest axis 2A to the inside of a graduated thin ring of brass, the shortest axis 2C coinciding also with a diameter of the ring. The ring was attached to the torsion-balance. We made use of a great horseshoe-electro-magnet placed vertically, and excited by twelve of Grove's elements. The diameter of the flat poles was about 4 inches, the distance between their centres 10·24. The ellipsoid was brought into such a position that its centre lay in the vertical plane passing through the centres of the two poles, at a distance of 30·74 inches from the point midway between them, nearly 4·1 inches below the horizontal plane touching both poles. A dipping-needle, having its centre similarly placed within the same vertical plane, and by means of a counterpoise pointing horizontally when the current was interrupted, pointed horizontally too when the current was closed.

When suspended with its longest axis 2A vertical, the ellipsoid set its mean axis 2B axially; when with its shortest axis 2C vertical, the same mean axis 2B pointed equatorially. Therefore, as the vertical axis of suspension passes within the principal plane AC, *i. e.* within the middle plane of the ring, from the first position to the second, there ought to be found a position where the mean axis 2B passes from the axial into the equatorial line. Accordingly we got the two magnetic axes of the ellipsoid, equally distant on both sides from the longest axis 2A, and two suspensions corresponding to each axis and correcting each other.

51. In the four suspensions along 25°, 155°, 205°, 335°, the torsion-wire being attached to the ring in the corresponding division, beginning from one extremity of the longest axis and going round from 0° to 360°, the mean axis 2B pointed axially, the longest axis 2A being within the equatorial plane. In the four suspensions along 30°, 150°, 210°, 330°, the mean axis 2B pointed equatorially, the longest axis 2A being within the axial plane; but here the directing power appeared not quite so strong as before, and most weak in the second and fourth of these suspensions. When suspended along $27\frac{1}{2}$°, $152\frac{1}{2}$°, $207\frac{1}{2}$°, $332\frac{1}{2}$° the ellipsoid set its mean axis 2B axially. Hence the angle, denoted before by ω, is between $27\frac{1}{2}$° and 30°, about 29°. Therefore the angle between the two sets of circular sections of the auxiliary ellipsoid, *i. e.* the angle between the two magnetic axes, is about 58°, while the two circular sections of the induced ellipsoid of iron are perpendicular to each other.

The same experiment has been repeated exactly in the same way with the smaller ellipsoid of iron. The resulting magnetic axis had nearly the same position as in the former case, the angle ω being fully 30° instead of 29°. This angle was not sensibly changed when the distance from the point midway between the centres of the flat poles of the electro-magnet was increased from 30·74 to 39·37 inches.

52. 2nd. Then we proceeded to a second series of experiments, in order, first, to verify the equation (30.), which may be considered as the test of Poisson's theory; secondly, to determine by means of the method of oscillations the same angle ω which we formerly obtained by direct observation. We made use of the smaller ellipsoid, attached without the divided ring by means of very fine copper wires to the silk thread of the torsion-balance, having its centre always in the same place, 39·37 inches distant from the equatorial plane of the electro-magnet. In three different suspensions the axes 2A, 2B, 2C were successively vertical, the corresponding principal planes, marked by white paint, oscillating horizontally, always at the same height. It had been ascertained that the copper wire did not sensibly increase the moment of inertia of the iron ellipsoid, nor had the torsion of the carefully selected silk thread any sensible influence on the number of oscillations. The current being interrupted, the longer axis of the horizontal section of the ellipsoid was, before oscillating, directed by means of the torsion-balance towards the poles. The observed numbers of oscillations corresponding to different magnetic powers were in full accordance. Here I shall refer only to the last series of observations, where the current was excited by twelve elements, the acids not having been used before, the zinc being newly amalgamated. Immediately after each observation we repeated the experiment, with this difference, that that extremity of the vertical axis which had pointed downwards was now directed upwards. Thus we got two numbers relating to each of the three cases. After having determined the second number of the first case, the ellipsoid was brought again into the primitive position: thus was obtained a third number equal to the first.

TABLE.

Number of oscillations.	Number of seconds required.	
	Axis 2B vertical.	
18	169, 170, 169	Mean 169·33
	Axis 2C vertical.	
14	159, 162	Mean 160·5
	Axis 2A vertical.	
12	161, 157	Mean 159.

To control the constancy of the current during the observations, in each suspension the number of seconds corresponding to the first and last half number of oscillations was separately marked; but the number of seconds having been found in all cases equal to half the total number, I thought it not necessary to note it here. Finally, the first observation was twice repeated; we found 166·5 instead of 169·5, indicating a small increase of the magnetic power.

The ratio of the three mean numbers of the Table being independent of the intensity of the current, the next day we determined by observation in a more peculiar way the ratio of the first and second number, and afterwards the ratio of the first and third. In both cases we got small differences. Accordingly, the observation of the first case admitting the greatest accuracy, we retained the first mean number 169·33; instead of the second, 160·5, we adopted 159·75; instead of the third, 159, we adopted 160·2.

The square of the time of one oscillation concluded from the corrected Table is in the three cases

$$\Theta_{//}^2 = 88\cdot51,$$
$$\Theta_{///}^2 = 130\cdot19,$$
$$\Theta_{/}^2 = 178\cdot22.$$

Putting

$$A^2=400, \quad B^2=160, \quad C^2=100,$$

we get

$$A^2+C^2=500, \quad A^2+B^2=560, \quad B^2+C^2=260;$$

therefore

$$\frac{A^2+C^2}{\Theta_{//}^2}=5\cdot649,$$
$$\frac{A^2+B^2}{\Theta_{///}^2}=4\cdot301,$$
$$\frac{B^2+C^2}{\Theta_{/}^2}=1\cdot459.$$

The sum of the last two numbers, equal to

$$5\cdot760,$$

differs from the first by only

$$0\cdot111,$$

i. e. $\frac{1}{50}$ nearly. This error falls within the limits of the errors of observation.

The equation (30.) being thus verified, we obtain, according to (28.),

$$\tan\omega=\sqrt{\frac{1459}{4301}},$$

whence

$$\omega=30^\circ\ 13'.$$

This value of the angle ω agrees very well with the value concluded, in a less exact way, from the first series of observations*.

III. *Theory of the magnetic induction of crystals, and its experimental verification.*

53. The results we obtained in the preceding section remain unchanged as long as the dimensions of the influenced body may be neglected, with regard to the distance of the pole, *i. e.* as long as within the influenced body the lines of magnetic force are sensibly parallel. The formulæ therefore we deduced from POISSON'S theory, relating to a finite ellipsoid influenced by an infinitely distant pole, may be immediately applied to the infinitely small particle of a crystal at a finite distance from the inducing pole. If the particle be placed between two opposite poles, we may substitute for the two poles a single one of double intensity.

54. Let us then conceive, as we did before, the crystal to consist of an infinite number of influenced small ellipsoids, not sensibly acting on each other. Every such ellipsoid will furnish a moment of rotation represented by

$$\frac{2\tan\xi d\varphi}{r^2},$$

ξ and r being determined by the auxiliary ellipsoid. The resulting moment of rotation will be represented by the integral

$$2\int\frac{\tan\xi d\varphi}{r^2},$$

extended to the entire mass of the crystal. If we admit that all particles of a crystallized mass *are of the same form and similarly directed*, ξ and r become constant, whence the resulting moment,

$$\frac{2\varphi\sin\xi}{r^2}.$$

* We may regard here the above-described experiments as a sufficient verification of the results emanating from POISSON'S theory, in the case of an ellipsoid of iron influenced by a distant pole. A more complete verification of this theory lies beyond the limits of this paper. The ellipsoid of iron may be replaced by a similar one of cobalt or nickel; according to theory, the angle (2ω) between the two axes will be found to be a different one. We may derive from experiment the value of POISSON'S constant k (art. 24, note), and compare the value of the magnetic induction with gravitation. Whatever may be the interest connected with these questions, they must be reserved to another series of experiments; the more so, as our horseshoe-electro-magnet—whose two poles induce a distant ellipsoid in opposite sense, along directions which are to be previously determined by observation—is, for such researches, to be replaced by a system of two cylindric electro-magnets having a common axis.

The moment therefore is the same, as if the crystal, whatever may be its exterior form, were transformed into an amorphous ellipsoid of the same mass.

55. A finite amorphous ellipsoid, attached to any vertical axis, when influenced by an infinitely distant pole falling within the horizontal plane, will be directed in the same way as a similar ellipsoid rotating round its vertical diameter. Having therefore a series of such ellipsoids attached one to another and rotating round a common axis, each of them will be forced into the same position of equilibrium as when rotating alone. Hence a crystal too, oscillating between the two poles, is *directed*, whatever may be its shape, like one of its ultimate particles, *i. e.* like the above-mentioned ellipsoid.

56. Therefore, in the case of the magnetic induction of crystals, the same analytical formulæ subsist, which, in the preceding section, we derived in the case of an influenced amorphous ellipsoid by means of auxiliary ellipsoids. All former formulæ concerning the direction of the crystal remain unchanged. With regard to the formulæ bearing upon the law of its oscillations, we shall suppose the oscillating crystal to be always of a spherical form; then the moment of inertia, corresponding to any vertical axis, is equal to $\frac{2}{5}MR^2$, denoting the mass of the crystal by M and its radius by R. Accordingly, the formulæ (26.), (28.), (29.), (30.) are to be replaced by

$$\Theta^2=\frac{MR^2\pi^2}{5\varphi(a'^2-b'^2)}=\frac{MR^2\pi^2}{5\varphi(a^2-c^2)}\cdot\frac{1}{\sin\psi\sin\psi'} \quad (33.)$$

$$\frac{\Theta_{\prime\prime}}{\Theta_{\prime}}=\sin\omega,\qquad \frac{\Theta_{\prime\prime}}{\Theta_{\prime\prime\prime}}=\cos\omega,\qquad \frac{\Theta_{\prime\prime\prime}}{\Theta_{\prime}}=\tan\omega. \quad (34.)$$

$$\Theta^2=\Theta_{\prime\prime}^2\sin\psi\sin\psi'. \quad (35.)$$

$$\frac{1}{\Theta_{\prime}^2}+\frac{1}{\Theta_{\prime\prime\prime}^2}=\frac{1}{\Theta_{\prime\prime}^2}. \quad (36.)$$

In all these formulæ, we suppose known, as we did before, the direction of the three axes $2a$, $2b$, $2c$ of the first ellipsoid, *i. e.* the direction of the axes of greatest, mean, and least induction. The angle between the two magnetic axes is always denoted by 2ω. The time of one oscillation is denoted by $\Theta_{\prime}$, $\Theta_{\prime\prime}$, $\Theta_{\prime\prime\prime}$, and $\Theta_{\prime}$ in the cases where the crystal, of a spherical form, successively oscillates round its axes of greatest, mean, and least induction, and round any diameter determined by the two angles ψ and ψ' between it and the two magnetic axes.

57. The first question we here meet with, is to determine for any crystalline substance the constants, especially ω, upon which depends the position of the crystal, when suspended between the poles, and the law of its oscillations.

1st. We may, as shown in the case of ferridcyanide of potassium, sulphate of zinc, and formiate of copper, find by experiment, within the crystal, the two magnetic axes, remembering that the crystal, when suspended along one of them, is not acted upon in an extraordinary way, not at all acted upon when its form is, for instance, that of a sphere.

2nd. Any one suspension of the crystal along a vertical axis, fixed with regard to the axes of induction, is sufficient to determine the angle ω, and hence the position of the two magnetic axes. We may for this purpose make use of the formula (9.), and any one of the formulæ (11.)–(13.), or of the formulæ (18.), (19.). Reciprocally, ω being determined, we may calculate the position of the crystal in any given suspension, by means of (9.) and (11)–(13.), or of (14.) and (15.).

3rd. Let the spherical crystal be successively suspended along any two diameters, previously determined within it with regard to its axis of induction, and let us take in both cases the number of oscillations it performs in a given time, when brought a little out of its position of stable equilibrium. From the two numbers thus obtained we can easily calculate the value of ω (34.), most easily when the crystal was suspended along any two of its axes of induction (33.).

58. By determining for any crystal the value of the angle ω in the three different ways, we are enabled to verify the theory we have put forward. Such a verification, however, is immediately supplied by the equation (36.), which may be expressed thus:—

The sum of the squares of the numbers of oscillations which a sphere turned out of a crystalline substance performs in a given time, when successively suspended along the greatest and least axis of induction, equals the square of the number of oscillations performed when suspended along the mean axis of induction.

59. Among the crystals I was able to provide, the most proper to be used in order to verify the proposed theory of the magnetic induction of crystals was formiate of copper, which I crystallized myself. After some trials, I succeeded in getting turned, by M. FESSEL, out of a fine crystal of this salt, a perfectly homogeneous sphere 0·39 of an inch in diameter. A great circle traced on its varnished surface marked the cleavage plane of the crystal. [In order to verify, after experiment, the position of this plane, the sphere was cloven by means of the heat produced in the focus of a burning lens.] The sphere was supported by a small circular ring of very thin mica, attached by three thin silk threads to the double cocoon-fibre of the torsion-balance. The distance of the pointed poles, the extremities of large iron pieces put on the flat poles of the large electro-magnet, from each other, was about 1·58 of an inch.

60. The sphere was first placed on the ring, with its cleavage plane horizontal, in order to determine the symmetrical plane, this plane being in our case vertical and forced by the power of the magnetic poles into the axial position. After having traced on the surface of the sphere the symmetrical plane thus obtained, we placed it on the ring, with this plane horizontal. The two points, marked on the new great circle, pointing axially and equatorially, indicated the direction of the greatest and least axis of magnetic induction, the mean axis being perpendicular to the symmetrical plane. The two principal planes perpendicular to the symmetrical plane were likewise marked on the surface of the sphere by great circles, one of which, passing through the least and mean axis, is nearly coincident with the cleavage plane, the angle between the two planes being nearly 3°.

61. 1st. We tried first to determine directly the position of the two magnetic axes within the symmetrical plane, by turning the sphere, when influenced by the poles, around the mean axis, the symmetrical plane remaining always vertical. But here we observed that the passage from the axial position of the symmetrical plane to the equa-

torial one takes place gradually, the crystal passing through all intermediate positions while it rotates through several degrees round the mean axis of rotation. This mode of determining ω admits therefore of no great accuracy.

The inclination of the cleavage plane to the horizontal plane was determined, when the symmetrical plane pointed 45°, bisecting the angle between the axial and the equatorial plane. We got for this angle, by inclining the cleavage plane on both sides, on one side 21°, on the other 26°, whence $\omega=23\frac{1}{2}°$, and the angle between the normal to the cleavage plane and the mean axis $2\frac{1}{2}°$. [Instead of $23\frac{1}{2}°$, we got formerly 25° (art. 21); instead of $2\frac{1}{2}°$, by direct measure 3°.]

62. 2nd. Then the number of small oscillations was counted which the same sphere performed in one second, when successively suspended along its three axes of induction. In one series of observations six, in another twelve of GROVE's elements were used. In both series the crystal was twice suspended along each axis. The following Table furnishes the numbers thus obtained:—

Suspended along	Numbers of oscillations.		
Six elements.			
The greatest axis	$22\frac{1}{2}$	23	$\frac{1}{\Theta_{I}}$
The mean axis	53	53	$\frac{1}{\Theta_{II}}$
The least axis	49	49	$\frac{1}{\Theta_{III}}$
Twelve elements.			
The greatest axis	31	$31\frac{1}{2}$	$\frac{1}{\Theta}$
The mean axis	73	73	$\frac{1}{\Theta_{II}}$
The least axis	67	67	$\frac{1}{\Theta_{III}}$

From the first series of observations we get

$$\frac{1}{\Theta^2_{I}}+\frac{1}{\Theta^2_{III}}=2918, \qquad \frac{1}{\Theta^2_{II}}=2809;$$

from the second,

$$\frac{1}{\Theta^2_{I}}+\frac{1}{\Theta^2_{III}}=5166, \qquad \frac{1}{\Theta^2_{II}}=5329.$$

The differences between the two numbers thus obtained in both cases (109 and −163), are so small, with regard to the limits of error, that the theorem of article 58 can be considered confirmed by experiment.

In order to eliminate these small differences, let us remember that the ratio of any two corresponding numbers in the two series of observations is to be the same, this ratio equaling the ratio of the inducing powers of the magnet. Again, in the two series of observations, the numbers corresponding to the same mode of suspension are likewise in the same ratio. These two conditions are fulfilled with regard to the suspensions along the greatest and mean axis. We get as near as possible

$$\frac{73}{53}=\frac{31}{22{\cdot}5}=1{\cdot}38,$$

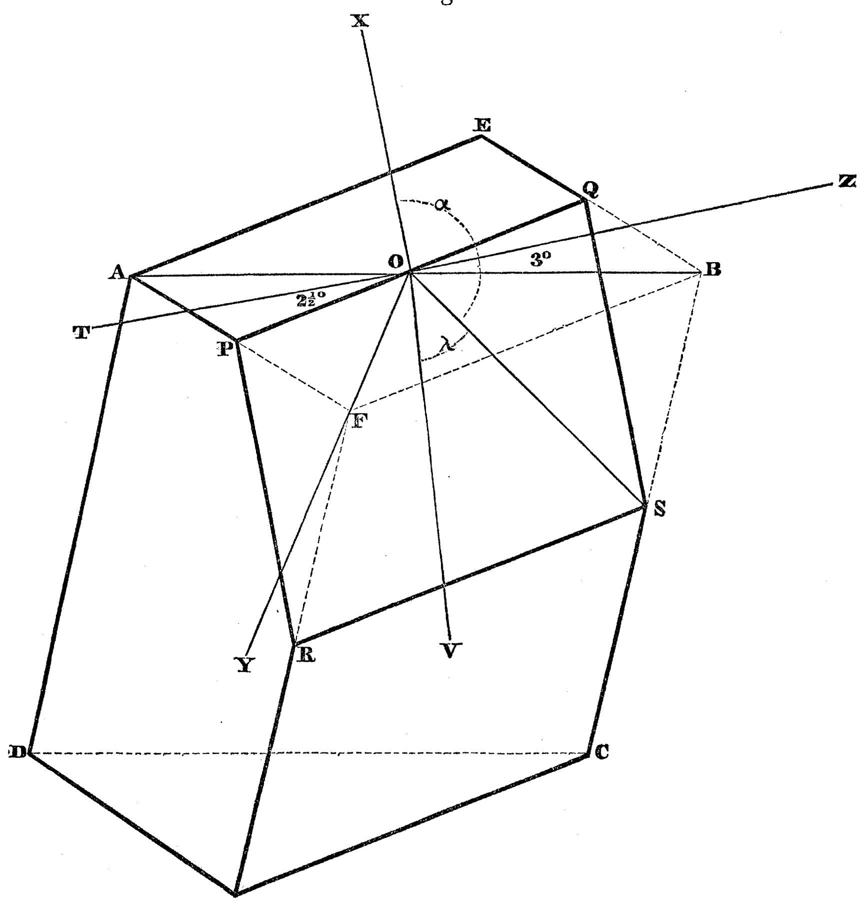
this number indicating the relative inducing power. We may regard, therefore, the four numbers 22·5, 31, 53, 73 as the most exact. Among all possible vertical axes of suspension, the *minimum* number of oscillations corresponds to a suspension along the greatest axis, the *maximum* number to a suspension along the mean axis. Therefore we are authorized to prefer 22·5 and 31 to 23 and 31·5. In conformity with that, we may, according to our theorem, calculate the numbers of oscillations in the third mode of suspension, which admits the greatest errors of observation. The two resulting numbers are 48·0 and 68·3, instead of 49 and 67. Finally, we obtain (34.),

$$\sin\omega=\frac{22{\cdot}5}{53}=\frac{31}{73},$$

whence $$\omega=25^\circ{\cdot}8.$$

63. 3rd. We shall deduce the value of ω from the observed position of an influenced

Fig. 26.



crystal, suspended along any known direction. The crystals I had obtained exhibiting in a very pronounced way several faces of the primitive octahedron, we suspended one of them with such a face horizontal, and were enabled to observe with accuracy the angle between the equatorial line and an edge of the primitive octahedron, the line of intersection of the horizontal face and the cleavage plane. This angle was found to be $2\frac{1}{2}°$. The same value of the angle was obtained as well by different natural crystals as by a circular plate turned out of a fine crystal, on whose base, lying in the face above mentioned, the direction of the edge was marked by a line, along which, moreover, was attached a long and very thin filament of glass, indicating more distinctly the pointing of the crystal.

Let ABDC (fig. 26) be the symmetrical plane containing the two axes of greatest and least induction OX, OZ; AEBF the cleavage plane perpendicular to the symmetrical plane, and containing the mean axis OY; and lastly, PQSR the horizontally-suspended face of the primitive octahedron, intersecting ABDC in OS and AEBF in the edge PQ. The equatorial and the axial line OT and OV lie in the horizontally-oscillating face PQSR; the measured angle POT equals $2\frac{1}{2}°$. According to M. HEUSSER's measures, the inclination of the face PQSR to the cleavage plane is 51° 31′, and from his above-quoted angles (art. 17) we derive QOB=44° 51′. Hence, considering the rectangular spherical triangle determined by OQ, OS, OB, we get

$$QOS=57° 59', \quad BOS=41° 35\tfrac{1}{2}', \quad \phi=56° 17',$$

denoting by ϕ the angle between the face in question and the symmetrical plane. The angle ZOB within the symmetrical plane being 3° (art. 19), we get

$$\alpha=XOS=BOS+93°=134° 35\tfrac{1}{2}',$$

and in like manner

$$\lambda=VOS=POT+90°-QOS=34° 31'.$$

Starting from these values of the angles ϕ, α and λ, we obtain, by means of the formulæ (9.) and (12.), or (18.) and (19.),

$$\omega=25° 3'*.$$

64. After having expounded the general theory of the magnetic induction of crystals,

* Hitherto we have only determined, by various methods, the value of the angle ω, and thus obtained a linear relation between the three axes of the ellipsoid of induction. If, by considerations exceeding the limits of this paper, we should be enabled to get the ratio itself of these axes, we could hence deduce the form of the magnetic particles of the examined crystal, supposing that, in fact, these particles are similar ellipsoids, similarly directed, all induced by the magnetic pole, but not sensibly inducing each other. Mr. W. THOMSON, however, has published a curious theorem, according to which a body of any exterior shape is influenced by an infinitely distant pole like a certain ellipsoid whose axes are to be determined in each case. Hence an infinite variety of forms corresponds to a known ellipsoid of induction. Again, each physical condition of crystals leading to our ellipsoid of magnetic induction has, to the present time, independently of POISSON's hypothetical views on matter, the same claim to be the law of nature. We may generally conceive an amorphous substance to consist of equal particles, pointing in all different directions, which by the act of crystallization become directed in the same way. But I think it probable we shall obtain the same results by admitting spherical particles, which, according to their different proximity along different directions within the crystal, will equally lead, by their mutual induction, to three axes of induction; thus each such particle may possibly be acted upon in the same way as our ellipsoidal particles.

we may easily apply it to the case where the auxiliary ellipsoids are *ellipsoids of revolution*, and therefore the two magnetic axes coincident along the same line. The crystals in this case are to be called *uniaxal*. We may distinguish too *positive* and *negative* uniaxal crystals. In positive crystals $a^2=b^2$, and therefore $\omega=0$; in negative crystals $b^2=c^2$, and therefore $\omega=90°$. The induction in both cases may be as well a diamagnetic as a paramagnetic one.

65. Since the two axes of the horizontal section of the first auxiliary ellipsoid point axially and equatorially, as they do in the general case, the position of a uniaxal crystal, when suspended between the two poles along any of its diameters, is immediately obtained. Here the axis of revolution, *i. e.* the magnetic axis of the crystal, when projected on the horizontal plane, is one of these two axes, the other being the intersecting line of the equatorial plane of the ellipsoid and the horizontal plane. If the crystal be positive, this intersecting line is coincident with the shorter axis of the elliptical section within the horizontal plane; if it be negative, with the longer axis. Hence a paramagnetic crystal, when positive, sets its magnetic axis in the axial, when negative in the equatorial plane of the electro-magnet. In diamagnetic crystals, when positive, the magnetic axis is forced into the equatorial, when negative into the axial plane. In order to give a true description of this fact, you may say, in all cases the magnetic axis of the crystal is either attracted or repelled by the poles.

This law can easily be verified in a most distinct way by a great number of paramagnetic and diamagnetic, positive and negative crystals. I think it therefore not necessary to refer here to new experiments.

66. In most cases the magnetic axis is known, if not, it can easily be found by suspending the crystal along any two vertical axes. Mark in both suspensions if the crystal be positive and paramagnetic, or negative and diamagnetic, on its surface the axial plane, if it be positive and diamagnetic, or negative and paramagnetic, the equatorial plane; in both cases the line of intersection within the crystal of the two planes marked on its surface is the magnetic axis.

67. The law of the small oscillations of the crystal, which again, for simplicity, we suppose of a spherical form, when suspended between the poles along any vertical axis, is represented by the former equations (33.) and (35.), now simplified thus:

$$\Theta=\frac{MR^2\pi^2}{5\varphi(a^2-c^2)}\cdot\frac{1}{\sin\psi^2} \quad \ldots \quad (37.)$$

$$\Theta_0=\Theta\sin\psi, \quad \ldots \quad (38.)$$

denoting by Θ the time of one oscillation, if the angle between the magnetic axis and the vertical axis of suspension equal ψ, and Θ_0 the time of one oscillation if $\psi=90°$, *i. e.* if the magnetic axis oscillate horizontally.

68. Remembering a former observation, according to which sulphate of iron, although belonging to the clinorhombic system, ranges among uniaxal crystals, having its magnetic axis within the symmetrical plane, I selected this salt to verify the last formula. I

provided a small but most homogeneous and transparent sphere, on whose varnished surface the symmetrical plane was marked by a great circle.

We succeeded first in establishing the former observation. The sphere of sulphate of iron was suspended between the two poles, as formerly the sphere of formiate of copper was, with this difference only, that, on account of the stronger paramagnetic induction, we made use of only two of GROVE'S elements instead of six. The sphere was first placed on the ring with the symmetrical plane horizontal, and the direction within it pointing axially, marked by a point on the corresponding great circle. The sphere being then placed on the ring, with the symmetrical plane vertical and the marked point situated in the vertical axis of suspension, did not move under the influence of the magnetic poles. Hence sulphate of iron is a uniaxal positive and paramagnetic crystal.

The sphere was then put on the ring successively with its symmetrical plane horizontal, and inclined 45° alternately on one side and the other.

The symmetrical plane.	Number of seconds required for twenty oscillations.	
Inclined +45°	60	
Horizontal		44
Inclined −45°	64	
Horizontal		45
Inclined +45°	61	
Horizontal		46
Inclined −45°	$65\frac{1}{2}$	
Horizontal		45
Inclined +45°	$63\frac{1}{2}$	
Mean numbers	$62 \cdot 8 = \Theta_0$	$45 = \Theta_0$

Hence

$$\frac{\Theta_0}{\Theta} = 0 \cdot 715 \quad \sin \psi = \sin 45° = 0 \cdot 707;$$

therefore, in this case also, the theory is fully confirmed by experiment.

69. Being desirous of examining in like manner a uniaxal and diamagnetic crystal, I selected a beautiful specimen of crystallized bismuth, showing the cleavage in the most perfect way. A sphere, 0·79 of an inch in diameter, was turned out of it, and on its surface was traced the equatorial circle, indicating the cleavage plane, and any meridional circle passing through the magnetic axis. But this sphere, on account of the electric currents excited within it, while it oscillates between the two poles, could not be subjected to experiment in the same way as the sphere of sulphate of iron. This is the case with regard to all substances enjoying a great conducting power, and not being magnetically influenced in so high a degree as iron, cobalt, and nickel.

70. Therefore, in order to verify the theory which has been expounded with regard to our sphere of crystallized bismuth, when suspended along its different diameters, we

were obliged to recur to the formula (24.), expressing the moment of rotation (L) round any vertical axis,

$$L=\varphi(a'^2-b'^2)\sin 2\vartheta=\varphi(a^2-c^2)\sin^2\psi\sin 2\vartheta. \quad . \quad . \quad . \quad . \quad . \quad (39.)$$

In this formula the angle between the vertical great circle of the sphere passing through the magnetic axis and the axial plane is denoted by ϑ, and the angle taken on the same great circle between the magnetic and the vertical axis by ψ. Denoting the value of the moment corresponding to $\psi=90°$, where the magnetic axis is vertical, by L_0, we obtain

$$L=L_0\sin^2\psi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40.)$$

The angle ψ remaining the same, the maximum of L corresponds to $\vartheta=45°$; denoting this maximum by L^0, we get

$$L=L^0\sin 2\vartheta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41.)$$

71. The sphere of crystallized bismuth was placed on a ring of very thin, not paramagnetic, copper wire, attached by three silk threads to the platinum wire of the torsion-balance. The centre of the sphere was brought into the middle point between the two poles, its marked meridional great circle remaining always vertical.

When the equatorial great circle was horizontal, there was no sensible direction; the torsion-wire being turned through any angle, the sphere rotated round its vertical axis through the same angle. When the equatorial circle was more and more inclined, the directing power emanating from the poles, *i. e.* the moment L, increased till the equatorial plane became vertical, and therefore the magnetic axis horizontal. Let us consider in a more special way this last case, corresponding to $\psi=90°$, and the case corresponding to $\psi=45°$. In all cases the marked meridional plane points axially: so it did in our two cases most exactly, after it had been brought by means of the torsion-balance into that position while the current was interrupted. The current being established, by turning the platinum wire the meridional circle was brought out of the axial position. If the number of degrees through which the wire rotated be δ, and the declination of the meridional plane of the sphere from the axial position ϑ, the number $\delta-\vartheta$ indicates the value of the moment of rotation in the corresponding position of the crystal. The number of degrees $(\delta-\vartheta)$ corresponding to different values of ψ, but to the same declination ϑ of the meridional plane of the crystal from the axial position, is proportional to $\sin^2\psi$. In our two cases these corresponding numbers are in the ratio of 2 : 1. It was confirmed by experiment.

72. While in the first suspension the torsion-wire was slowly turned, the angle of declination more and more increased; but finally, just when it reached 45°, corresponding to a certain value of δ, the crystalline sphere was suddenly driven into a nearly opposite position of stable equilibrium. A similar reversion took place after a rotation of the wire of the torsion-balance in the same sense through 180° more, and so on. In our second case, we got the same phenomena, with this difference only; that the first reversion took place when the torsion-wire was turned through a smaller number of degrees. In both cases we made use of twelve of Grove's elements.

In the first case, the value of δ, in the moment when the first reversion took place, in two successive observations was found to be

$$\left.\begin{matrix}561\\563\end{matrix}\right\}\text{Mean } 562;$$

in the second case, we obtained

$$\left.\begin{matrix}295\\300\end{matrix}\right\}\text{Mean } 297{\cdot}5;$$

whence

$$\frac{297{\cdot}5-45}{562-45}=\frac{252{\cdot}5}{517}=0{\cdot}498,$$

consequently very near the value of $\sin^2\psi=0{\cdot}5$.

73. The declination (ϑ') of the meridional plane, after the suspension of the crystal, may easily be calculated. Let the number of degrees through which the meridional plane rotates, when passing from the position of unstable equilibrium, corresponding to $\vartheta=45°$, into the new position of stable equilibrium, be denoted by z, whence $\vartheta'=z+45$. Then, according to (41.), we get

$$\sin 2(z-135)=\frac{\delta-45-z}{\delta-45},$$

whence

$$\sin^2 z=\frac{z}{2(\delta-45)}. \qquad (42.)$$

In our first case, where $\delta-45=517$, we obtain

$$z=157{\cdot}05, \quad \vartheta'=180°+22°\ 3';$$

in the second case, where $\delta-45=252{\cdot}5$,

$$z=147{\cdot}30, \quad \vartheta'=180°+12°\ 18'.$$

These values, according to an approximative estimation, agree with observation*.

* The same mode of experimenting may be applied to the general case of biaxal crystals, by substituting for the formula (39.),

$$L=\phi(a^2-c^2)\sin\psi\sin\psi'\sin 2\vartheta;$$

denoting, as we did before, the angles between the vertical and the two magnetic axes by ψ and ψ'. When the vertical axis successively coincides with the axes of greatest, mean, and least induction, the corresponding moments of rotation are

$$L_{\prime}=\phi(a^2-c^2)\sin^2\omega\sin 2\vartheta,$$
$$L_{\prime\prime}=\phi(a^2-c^2)\sin 2\vartheta,$$
$$L_{\prime\prime\prime}=\phi(a^2-c^2)\cos^2\omega\sin 2\vartheta;$$

whence, corresponding to any value of ϑ,

$$L_{\prime}:L_{\prime\prime}:L_{\prime\prime\prime}=\sin^2\omega:1:\cos^2\omega.$$

Hence we may determine by experiment the position of the two magnetic axes of a crystal in the following new way, allowing of great accuracy. Attach the crystal, after having given to it the form of a sphere, to the wire of the torsion-balance, and let there be no torsion when the sphere influenced by the poles takes a certain direction. Let, successively, the axis of greatest, mean, and least induction be vertical, and determine in each suspension the number of degrees through which the torsion-wire, slowly turned, rotates till the reversion of the sphere takes place. *The sum of the numbers thus obtained in the first and the third case equals the number obtained in the second case.* The first number divided by the second equals $\sin^2\omega$; the third by the second, $\cos^2\omega$; the first by the third, $\tan^2\omega$.

IV. *Position of the three axes of magnetic induction with regard to the primitive form, in crystals belonging to different systems.*

74. A great number of crystals, the finest of which were supplied by the extreme liberality of Professor Böttger of Frankfort, have been examined by myself in connexion with Professor Beer, in order to establish their optic as well as magnetic properties, with regard to their crystalline form. The results are published in two papers in Poggendorff's 'Annalen.' A third paper, ready to be sent to press nearly two years ago, did not appear, since, according to the newly-adopted theory, I found it necessary to present the results obtained by experiment in a different way. From these papers are taken nearly all the observations referred to in this section.

75. In crystals having *no* plane of symmetry, there is not the least indication of the position of the three unequal axes of magnetic induction. In no case has the position of these axes been determined.

Sulphate of copper is a crystal of this kind, strongly exhibiting the extraordinary action. I provided a sphere, turned out of a large crystal, in order to determine the position of the three axes of induction, and hence the two magnetic axes. According to art. 39, I intended to trace on its surface the two curves indicating the diameters which point axially and equatorially, when the sphere, suspended between the two poles, rotates successively round any two horizontal diameters. The practicability of this proceeding had been previously proved in the case of crystals belonging to a different system; but here we did not succeed, on account of the want of homogeneity of the sphere of sulphate of copper, which in its different parts contained different quantities of sulphate of iron *.

Cyanite furnishes another instance of extraordinary paramagnetic induction. Strongly paramagnetic prisms of this mineral show the extraordinary action, when suspended horizontally, in a sensible way, *even under the magnetic induction of the earth.* Rotating round their horizontal axis, they point towards different azimuths.

Bichromate of potash shows very distinctly the extraordinary paramagnetic, racemic acid the extraordinary diamagnetic induction.

76. The crystals *having a single plane of symmetry* may be referred to an oblique prism with a rhombic base, the plane of symmetry passing through one of the diagonals of the base. Two axes of paramagnetic induction always lie in the plane of symmetry, the remaining third one being perpendicular to it. When any crystal of this system is to be examined, you may suspend it first along the line perpendicular to the

* All specimens of copper, except of English, which I had the opportunity to examine, were found to be strongly paramagnetic, even such specimens as were obtained by galvano-plastic deposition; so were all salts of its oxides. I was first inclined to attribute the paramagnetic condition in both cases to unmixed iron; but on closer examination, I convinced myself that oxide of copper is paramagnetic *per se*, and so are all its salts; in like manner as copper, dioxide of copper and its salts (sulphite of copper and ammonia, $\text{Ɖu}\ddot{S}+\dot{Am}\ddot{S}$) are diamagnetic.

symmetrical plane, and mark within this plane the directions both axial and equatorial. These two directions, representing the two axes of induction within the symmetrical plane, are not at all indicated by the crystalline form; they are to be found by experiment. The crystal, when successively suspended along each of the three axes of paramagnetic induction, sets one of these axes twice axially, another one twice equatorially. In the case of paramagnetic induction, the first axis is the longest, the second the shortest. In the case of diamagnetic induction, the second axis, pointing twice equatorially, is the longest, the first the shortest one. In both cases, the two magnetic axes lie in the plane passing through these two axes of induction. Therefore the plane containing the two magnetic axes is either coincident with the symmetrical plane, or is perpendicular to it. We may distinguish three different cases, whether the crystal be paramagnetic or diamagnetic: 1. the axis of greatest, 2. the axis of least, 3. the axis of mean magnetic induction is perpendicular to the plane of symmetry. In the first two cases, the plane containing the two magnetic axes is perpendicular to the symmetrical plane, in the third case it falls within it.

	Paramagnetic crystals.	Diamagnetic crystals.
First case . .	Diopside.	Hyposulphite of soda ($\dot{Na}\ddot{S}$).
	Red ferridcyanide of potassium.	
Second case. .	. . . .	Acetate of soda.
	. . . .	Acetate of lead.
Third case . .	Formiate of copper.	
	Acetate of copper.	

77. The crystals whose primitive form can be referred to a right prism with a rhombic base have *three planes of symmetry*, intersecting each other along the three axes of magnetic induction. We can, as we did in the former case, determine, by suspending the crystal successively along each of these axes, which of them is the longest, mean, and shortest. The two magnetic axes, always to be sought within the plane of the longest and the shortest axis of induction, lie in any one of the three planes of symmetry. We may distinguish six different cases, as well of paramagnetic as diamagnetic induction.

We shall denote, as we did in the first section, the axis of the right prism, the shorter and the longer diagonal of its base, by α, $\varkappa$, λ, the longest axis of the ellipsoid of induction (art. 47.) being always a^2, the mean b^2, and the shortest c^2. The six cases above mentioned may then be enumerated thus:—

	a^2	b^2	c^2	
	falling within			
1.	$\varkappa$	α	λ	paramagnetic, Sulphate of nickel.
				. . . . , Sulphate of nickel and zinc.
				diamagnetic, SEIGNETTE's salt.
2.	λ	α	$\varkappa$	

3.	α	$\varkappa$	λ	diamagnetic,	Aragonite.
				. . . . . . ,	Sulphate of zinc.
4.	λ	$\varkappa$	α	paramagnetic,	Staurolite.
				. . . . ,	Ferridcyanide of lead (3PbCy+FeCy³).
				. . . . ,	Sulphate of zinc and iron.
				. . . . ,	Sulphate of magnesia and iron.
				diamagnetic,	Anhydrite.
				. . . . . . ,	Hyposulphate of soda ($\dot{N}a\dddot{S}$).
5.	α	λ	$\varkappa$		
6.	$\varkappa$	λ	α	paramagnetic,	Ferridcyanide of potassium.
				diamagnetic,	Sulphur.
				. . . . . . ,	Citric acid.

In the cases 1 and 2, the two magnetic axes lie in the base of the right prism; in the cases 3 and 4, in the plane passing through its axis and the longer diagonal of its base; in the cases 5 and 6, in the plane passing through its axis and the shorter diagonal of its base.

In all cases, either the acute or the obtuse angle between the two magnetic axes is bisected by the axis of greatest magnetic induction. We shall count the angle 2ω from 0° to 180°, this angle being always bisected by the axis of greatest induction. [If this angle be counted up to 90° only, we may, analogously as in optics, according as the acute angle between the magnetic axes is bisected by the axis of greatest or by the axis of least induction, call the crystals magnetically positive or negative.]

78. In crystals having *one straight line of symmetry*—primitive form: rhombohedron, prism with a hexagonal base, prism with a square base—the ellipsoid of induction becomes a spheroid whose axis of revolution coincides with the line of symmetry. This line is likewise the magnetic axis. We meet in each of the two cases of magnetic induction with two classes of crystals. Here the crystals of the first class shall be called *positive*, of the second class, *negative*. In positive crystals the line of symmetry coincides with the axis of greatest paramagnetic or diamagnetic induction, in negative crystals with the axis of least induction. A uniaxal crystal, when suspended along any vertical axis, sets, if paramagnetic and positive or diamagnetic and negative, its magnetic axis in the axial; if paramagnetic and negative or diamagnetic and positive, in the equatorial plane.

Paramagnetic crystals.

Positive ($a^2=b^2$).	Negative ($b^2=c^2$).
Carbonate of iron.	Tourmaline.
Scapolite.	Beryl.
Green uranite.	Dioptase.
Sulphate of copper and calcium.	Vesuvian.
Sulphate of magnesia (containing iron).	Sulphate of nickel.
	Chloride of ammonium and copper ($AmCl+CuCl+2aq$).

Diamagnetic crystals.

Positive ($a^2=b^2$).	Negative ($b^2=c^2$).
Calcareous spar.	Bismuth.
Antimony.	Arsenic.
Molybdate of lead.	Ice.
Arseniate of lead.	Zircon.
Sulphate of potash.	Honeystone.
Nitrate of soda.	Cyanide of mercury.
	Arseniate of ammonia.

79. In all crystals belonging to the *tesseral* system, the ellipsoid of magnetic induction is reduced to a sphere, its three axes being equal to each other. There is *no* extraordinary action.

80. In different biaxal crystals the angle (2ω) between the two magnetic axes passes through all degrees from 0° to 180°. It is about 90° in sulphate of zinc. It is small in staurolite, near 180° in SEIGNETTE'S salt. In both cases the two magnetic axes approach to the longer diagonal of the base of the primitive right prism; in the first case the difference between the greatest and mean, in the second case the difference between the mean and least axis of induction is small. Along the longer diagonal the paramagnetic induction of staurolite is greatest, the diamagnetic induction of SEIGNETTE'S salt least. The plane containing the two axes has been determined by observation; it passes in the first case through the axis of the prism, in the second case it coincides with its base.

But if, in the case of three unequal axes of induction, the difference between any two of these axes becomes too small, the crystal, when examined between the poles, will exhibit the appearances of a crystal with only one magnetic axis. Crystals of this kind are the following ones:—Sulphate of iron, succinic acid, borax, cyanide of nickel and potassium.

Sulphate of iron, showing very strongly the extraordinary paramagnetic induction, may be regarded as a positive uniaxal crystal; its magnetic axis coincides with FRESNEL'S axis of greatest elasticity, being, within the plane of symmetry, inclined 75° to the cleavage plane.

Succinic acid, showing strongly the extraordinary diamagnetic induction, may be regarded as a positive uniaxal crystal, whose magnetic axis coincides likewise with FRESNEL'S axis of greatest elasticity.

Borax may be regarded as a diamagnetic negative crystal; its magnetic axis, coinciding with FRESNEL'S axis of least elasticity, is perpendicular to the plane of symmetry (the cleavage plane).

Cyanide of nickel and potassium also ranges among diamagnetic negative crystals; its magnetic axis coincides with FRESNEL'S axis of least elasticity, and accordingly lies in the plane of symmetry.

81. A crystal not belonging to the tesseral system will nevertheless show no extraordinary magnetic action, if all its three axes of induction are nearly equal to one

another. This is the case in the following crystals:—Oxide of iron (Eisenglanz), yellow ferrocyanide of potassium, cyanide of copper and potassium, quartz, sulphate of potash (whose primitive form is a rhombic prism), and topaz.

Note added during the printing of the paper.

The analogy between the phenomena presented by crystals in the case of light transmitted through them, and in the case of magnetic induction, may be fully explained by the circumstance that in both cases the laws of these phenomena depend upon an auxiliary ellipsoid. The diameters of the *optic* auxiliary ellipsoid represent the values of the reciprocal of the elasticity of the ether within the crystal; two of its axes coincide with the directions of the greatest and the least elasticity, the third and mean axis being perpendicular to these. On the other hand, the axes of the *magnetic* auxiliary ellipsoid are directed along the lines of the greatest, the least, and the mean magnetic induction, and their lengths represent the values of the reciprocal of these three inductions. This analogy becomes the more perspicuous and striking by the fact, that in the first case the *optic axes*, i. e. the two directions along which there is no double refraction, are perpendicular to the circular sections of the optic auxiliary ellipsoid; while in the second case, the *magnetic axes*, i. e. two lines such that the crystal on being suspended along either of them, between the two magnetic poles, is not acted upon by these poles in any extraordinary way, are perpendicular to the circular sections of the magnetic auxiliary ellipsoid.

In crystals with only one principal crystallographic axis, both auxiliary ellipsoids, the optic and the magnetic, become ellipsoids of revolution whose principal axis coincides with the crystallographic axis. In crystals whose primitive form is a right prism with a rhombic base, the three axes of both auxiliary ellipsoids are directed along the three principal crystallographic axes, but in both cases there is no indication at all given about the relative length of the three axes. In crystals belonging to the monoclinic system there is only one axis common to both auxiliary ellipsoids, this axis being perpendicular to the symmetric plane; the two remaining axes of both ellipsoids lie in this plane, where their position, not indicated by any general law, may be easily found by observation. In triclinic crystals there is no indication whatever given by the crystallographic form, regarding the position of the axes of the two ellipsoids, and therefore the determination of these axes is more difficult.

When a plane luminous wave is transmitted through a crystal along any direction, the vibrations in the front of the wave take place along the two axes of the ellipse, in which the optic auxiliary ellipsoid is intersected by the front. Experimentally these directions are determined by putting a plate of the crystal, bounded by faces parallel to the front of the wave, on a polarizing apparatus, and by turning it till it appears dark. Then one of the two directions of vibration lies in the primitive plane of polarization, the second is perpendicular to it. On the other hand, when a crystal is perpendicularly suspended between the two magnetic poles along any diameter of the magnetic auxiliary ellipsoid, one of the two axes of the horizontal elliptical section of this ellipsoid will point axially, the other equatorially. FRESNEL showed already that, by a simple geometrical construction, the two directions of vibration of any plane luminous wave may be deduced from the position of the two optic axes. I showed in the preceding paper and proved it by observation, that exactly the same construction gives the position of a crystal freely oscillating between the two poles of a magnet; the two horizontal lines pointing axially and equatorially are immediately obtained by projecting the two magnetic axes on the horizontal plane, and by bisecting the angle between these projections. In treating the inverse problem, "to find the magnetic axes of a crystal after having determined by observation the position which the crystal takes between the two poles, when suspended along any known direction," I proposed trigonometrical formulæ, which may be generalized in the following manner. Suppose OX, OY, OZ to be the three principal axes of induction, without knowing their relative values; let, within the suspended crystal, MON be the horizontal plane passing through the centre O and intersecting the principal plane OZ along OM; let ON be, within the horizontal plane, the line pointing either axially or equatorially. Denote by ϕ the angle between the planes MON and XOZ, by α and λ the angles MOX and NOM. Thus the position, within the crystal, of the horizontal plane is determined by the angles ϕ and α, the position of the crystal by the angle λ. After having determined, by means of the following two equations,

$$\tan(\eta+\alpha) = -\frac{\tan\lambda}{\cos\phi},$$

$$\tan(\eta'+\alpha) = \frac{\cos\lambda}{\cos\phi},$$

the two auxiliary angles η and η', calculate the value of the expression $\tan\eta\tan\eta'$. This value may be found to be either positive or negative; if negative, the absolute value may be either <1 or >1. Thus we obtain *three* different cases.

The position of the two magnetic axes is fixed by knowing the principal plane containing both axes, and the angle between each of them and a given one of the two axes of magnetic induction lying in the same plane. Denoting this angle by ω, we have, in the *first* case,

$$\tan\eta\tan\eta' = \tan^2\omega.$$

Both axes lie in the plane XOZ, ω being the angle between each of them and the axis OX. The last formula holds in both cases; the axis OX may be the axis of the greatest or of the least induction. From this axis the angle ω is to be measured.

In the *second* of the three above-mentioned cases, we obtain

$$\tan\eta\tan\eta' = -\sin^2\omega.$$

The two magnetic axes lie in the plane XOY, perpendicular to the plane XOZ, and intersecting this plane along OX, which may be the axis of greatest or least induction. The angle ω is to be measured from OX.

In the *third* case we have

$$\tan\eta\tan\eta' = -\frac{1}{\sin^2\omega}.$$

The two axes are situated in the plane ZOY perpendicular to the plane XOZ, and intersecting this plane along OZ, which may be the axis of the greatest or least induction. The angle ω is to be measured from OZ.

In the preceding paper I gave only the formulæ of the first case, where I supposed OX to be the axis of greatest induction. From this formula the others are easily deduced.

The magnetic axes of a crystal, either paramagnetic or diamagnetic, not examined before, whose primitive form is a right prism with a rhombic base, may, for instance, immediately be found by means of these formulæ. In this case, let the axes OX, OY and OZ coincide with the crystallographic axes in any order whatever; cut by parallel planes, whose inclination to these axes is known, a plate out of the crystal; let this plate oscillate horizontally between the two poles of the magnet, and mark on its surface a line pointing either axially or equatorially. Thus the angles ϕ, α and λ being determined, η and η', as well as $\tan\eta'\tan\eta'$, can be easily calculated. The sign and the value of the last expression shows by which of the last three equations the angle ω is to be calculated, and immediately indicates in which of the three principal planes the two magnetic axes are situated.

The position of the optic axes may be found *by the very same formulæ*. If we make use of the former plate, the angles ϕ and α remain the same, the angle λ only varies, and is to be determined by means of a convenient polarizing apparatus.

The optic axes relative to different colours being dispersed in the same principal plane, and even passing in certain cases from one principal plane to another, we cannot be surprised that the optic and magnetic axes are differently directed, and placed either in the same or in different planes. In ferridcyanide of potassium, for instance, the axis of the right prism is the axis of the greatest optic elasticity and of the least magnetic induction; the longer diagonal of the base is the axis of the least elasticity and mean induction; the shorter diagonal the axis of the mean elasticity and greatest induction. Therefore the two principal planes, containing the optic and the magnetic axes, intersect each other along the axis of the prism. In the case of sulphate of zinc, the axis of the prism is the axis of the mean optic elasticity and of the greatest diamagnetic induction; the longer diagonal of the rhombic base is the axis of the least elasticity as well as of the least induction; the shorter diagonal is the axis of the greatest elasticity and the mean induction. Accordingly the two optic axes lie in the base of the prism, the two magnetic axes in the plane passing through the axis of the prism and the longer diagonal of its base. In the case of formiate of copper, the mean axis of paramagnetic induction and that of optic elasticity are both perpendicular to the symmetric plane of the crystal; therefore the two optic axes and the two magnetic ones lie in this plane.

By turning, within their plane, each of the two optic axes for violet light in the same direction, these axes will pass into the position of the axes for red light, and afterwards, by turning still in the same direction, into the position of the magnetic axes. In common with Professor BEER, I examined a great number of crystals, in order to find a general law between the position of the magnetic axes of a crystal and its optic axes for the different colours, but without a satisfactory result. The laws mentioned in the second paragraph of these additional pages, by means of which we can in the simpler cases deduce from the primitive form of a crystal the position of the axes of both auxiliary ellipsoids, the magnetic and the optic—and also of all such auxiliary ellipsoids regarding, for instance, molecular elasticity, conduction of heat and electricity—are all we know.

I added this note to my original paper, in order to explain more distinctly than I did before the analogy between the optic and magnetic axes, which guided me during the different stages of my researches on the magnetic induction of crystals. My intention was not at all to enter into the mathematical solution of the great physical problem. All my experimental researches follow from the mere fact, that there exists an auxiliary ellipsoid of magnetic induction. The supposition of molecular ellipsoids of isotropic, paramagnetic or diamagnetic matter, is to be regarded as a means of discovering the mathematical laws to which the couple, tending to turn round a magnecrystal in a uniform field, was subject, and not as establishing these laws on a basis of molecular physics. Such a basis is to be obtained by Professor W. THOMSON'S theory only, which I highly regret not to have known when I wrote my paper. From this theory too, an abstract of which appeared in the Philosophical Magazine (March 1851), the above-mentioned auxiliary ellipsoid follows. Theoretically speaking, it was wrong to replace it by POISSON'S auxiliary ellipsoid, but this mistake does not in the least way affect the object of the paper presented to the Royal Society.